

## Quiz 8

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

**Question 1.** §3.5 #10 (2 marks) Find the area of the parallelogram determined by the given vectors  $\mathbf{u}$  and  $\mathbf{v}$ .

$$\mathbf{u} = (1, 1, 1) \quad \mathbf{v} = (3, 2, -5)$$

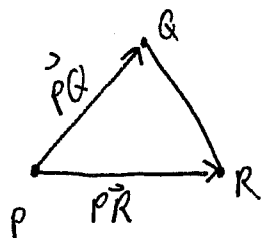
$$\text{Area} = \|\vec{u} \times \vec{v}\| = \|(-7, 8, -1)\| = \sqrt{(-7)^2 + 8^2 + (-1)^2} = \sqrt{49 + 64 + 1} = \sqrt{114}$$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ 3 & 2 & -5 \end{vmatrix} = \begin{vmatrix} 1 & 2 \\ 1 & -5 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 1 & 3 \\ 1 & -5 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix} \mathbf{k}$$

$$= (-7, 8, -1)$$

**Question 2.** §3.5 #16 (4 marks) Find the area of the triangle in 3-space that has the given vertices

$$P(1, -1, 2) \quad Q(0, 3, 4) \quad R(6, 1, 8)$$



$$\begin{aligned} \vec{PQ} &= Q - P \\ &= (0, 3, 4) - (1, -1, 2) \\ &= (-1, 4, 2) \\ \vec{PR} &= R - P \\ &= (6, 1, 8) - (1, -1, 2) \\ &= (5, 2, 6) \end{aligned}$$

$$\text{Area} = \frac{\|\vec{PQ} \times \vec{PR}\|}{2} = \frac{\sqrt{(20)^2 + (16)^2 + (-22)^2}}{2} = \frac{\sqrt{285}}{2}$$

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 4 & 2 \\ 5 & 2 & 6 \end{vmatrix} = (20, 16, -22)$$

**Question 3.** §3.5 #18 (4 marks) Find the volume of the parallelepiped with sides  $\mathbf{u}$ ,  $\mathbf{v}$ ,  $\mathbf{w}$ .

$$\mathbf{u} = (3, 1, 2) \quad \mathbf{v} = (4, 5, 1) \quad \mathbf{w} = (1, 2, 4)$$

$$\begin{aligned} V &= \left| \begin{vmatrix} 3 & 1 & 2 \\ 4 & 5 & 1 \\ 1 & 2 & 4 \end{vmatrix} \right| = \left| 3(-1)^{1+1} \begin{vmatrix} 5 & 1 \\ 2 & 4 \end{vmatrix} + 1(-1)^{1+2} \begin{vmatrix} 4 & 1 \\ 1 & 4 \end{vmatrix} + 2(-1)^{1+3} \begin{vmatrix} 4 & 5 \\ 1 & 2 \end{vmatrix} \right| \\ &= \left| 3[5(4) - (1)(2)] - [(4)(4) - (1)(1)] + 2[4(2) - 5(1)] \right| \\ &= \left| 3[18] - [15] + 2[3] \right| \\ &= \left| 54 - 15 + 6 \right| \\ &= 45 \end{aligned}$$