**Dawson College**: **Linear Algebra**: 201-105-dw-S04: Fall 2012

**Name:** ____________________
**Student ID:** ____________________

# Test 1

This test is graded out of 55 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

**Question 1.** *(10 marks)* Solve the following system by Gauss-Jordan elimination:

$$\begin{array}{rcrcrcrcl} 3x_1 & - & x_2 & + & x_3 & - & 3x_4 & = & 3 \\ 4x_1 & + & 3x_2 & - & x_3 & + & x_4 & = & 5 \\ 7x_1 & + & 2x_2 & & & - & 2x_4 & = & 8 \\ & & & & x_3 & - & x_4 & = & 1 \end{array}$$

**Question 2.** Consider the matrices:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 2 & 3 & 4 \end{bmatrix}, \; B = \begin{bmatrix} 2 & 2 & 1 \\ 0 & -4 & 0 \end{bmatrix} \; C = \begin{bmatrix} 0 & 1 \\ -3 & 0 \\ 3 & -1 \end{bmatrix} \; D = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix}$$

a. *(2 marks)* Compute the following, if possible.

$$C - D$$

b. *(2 marks)* Compute the following, if possible.

$$BC - D$$

c. *(2 marks)* Compute the following, if possible.

$$tr(C)$$

d. *(2 marks)* Compute the following, if possible.

$$tr(A)$$

e. *(3 marks)* Compute the following, if possible.

$$C^t B^t$$

f. *(2 marks)* Compute the following, if possible.

$$AB$$

g. *(5 marks)* Find $E$, if possible.

$$(I - DE)^{-1} = (BC)^t$$

**Question 3.** *(5 marks)* Given the following augmented matrix in row-echelon form, solve the system using back substitution.

$$\begin{bmatrix} 1 & 3 & -3 & 1 & 4 & 3 \\ 0 & 0 & 1 & -2 & 3 & -2 \\ 0 & 0 & 0 & 1 & 2 & 4 \end{bmatrix}$$

**Question 4.** *(5 marks)* Express

$$A = \begin{bmatrix} 3 & 4 \\ 5 & -2 \end{bmatrix} B = \begin{bmatrix} 3 & 4 \\ 8 & 2 \end{bmatrix}, C = \begin{bmatrix} 3 & 4 \\ 4 & 1 \end{bmatrix}$$

Find the elementary matrices $E_1$ and $E_2$ (if possible) such that

a. *(2 marks)* $E_1 A = B$

b. *(2 marks)* $E_2 B = C$

c. *(2 marks)* Are the matrices $A$ and $C$ row equivalent, justify?

**Question 5.** *(5 marks)* Find the inverse of

$$A = \begin{bmatrix} 2 & 3 & 0 \\ 3 & 4 & 0 \\ 0 & -6 & 1 \end{bmatrix}$$

**Question 6.** A square matrix $A$ is said to be idempotent if $A^2 = A$.

a. *(2 marks)* Show that if $A$ is idempotent, then so is $I - A$.

b. *(3 marks)* Show that if $A$ is idempotent, then $2A - I$ is invertible and is its own inverse.

**Question 7.** Consider the following system:

$$\begin{array}{ccccccc} x & + & y & + & 7z & = & -7 \\ & & 3y & + & 17z & = & -16 \\ & & & & a(a^2+1)z & = & 3a+3 \end{array}$$

where $a \in \mathbb{R}$, determine the values of $a$ so that the system has

a. *(2 marks)* a unique solution, justify.

b. *(2 marks)* infitely many solutions, justify.

c. *(2 marks)* no solutions, justify.

**Bonus Question.** *(5 marks)* Show that the following nonlinear system has 18 solutions if $0 \le \alpha \le 2\pi$, $0 \le \beta \le 2\pi$, and $0 \le \gamma \le 2\pi$.

$$\begin{array}{rcrcrcl} \sin\alpha & + & 2\cos\beta & + & 3\tan\gamma & = & 0 \\ 2\sin\alpha & + & 5\cos\beta & + & 3\tan\gamma & = & 0 \\ -\sin\alpha & - & 5\cos\beta & + & 5\tan\gamma & = & 0 \end{array}$$