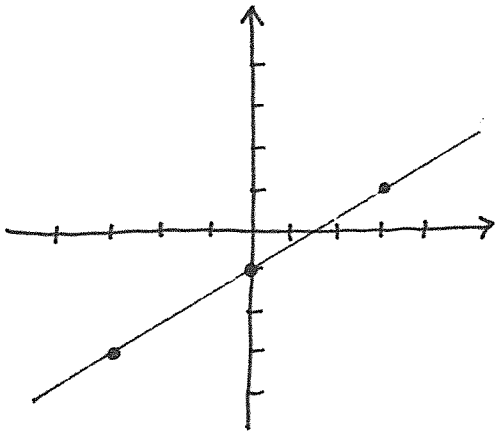
Assignment # 2 Solutions
201-943-DW section 1
Prof. Émilie Richer

Section 5.2.

34 marks

#8 $m = \frac{\Delta y}{\Delta x} = \frac{10-(-2)}{2-(-1)} = \frac{12}{3} = \underline{4}$

#18 Sketch $m = 2/3$ $(0,-1)$

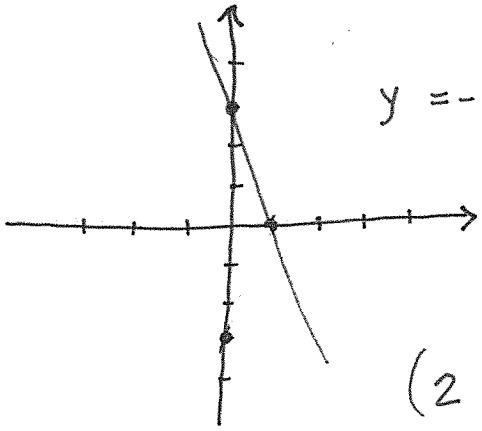


$y = \frac{2}{3}x - 1$

#30 $3x + y = 3$

x-intercept $y = 0$
$3x = 3$
$x = 1$
$(1,0)$

y-intercept $x = 0$
$y = 3$
$(0,3)$



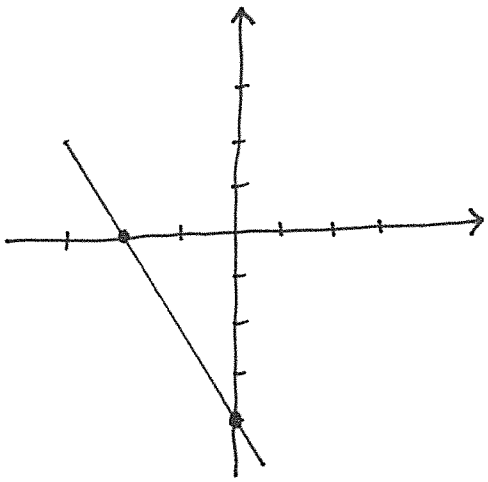
$y = -3x + 3$

(2 marks)

#34 $y = -2x - 4$

x-intercept
$x = 0$ $y = -4$
$(0,-4)$

y-intercept
$y = 0$ $0 = -2x - 4$
$4 = -2x$
$x = -2$
$(-2,0)$



(2 marks)

Section 5.4

#28 $1+6Q = 5p$ ①

$3p-4Q = 7$ ②

$\Longrightarrow$ Isolate Q in ①

$$Q = \frac{5p-1}{6}$$

$\Longrightarrow$ Replace in ②

$$3p - 4\left(\frac{5p-1}{6}\right) = 7$$

$$3p - \frac{20}{6}p + \frac{4}{6} = 7$$

multiply both sides of the equation by 6

$$18p - 20p + 4 = 42$$

$$-2p = 38$$

$$p = -19$$

$\Longrightarrow$ substitute $p=-19$ in $Q = \frac{5p-1}{6}$

$$= \frac{5(-19)-1}{6}$$

$$= -16$$

solution $\underline{(p,Q) = (-19,-16)}$

#54

40% soln + 85% soln = 20L of 60% soln

$$x + y = 20$$

$$0.4x + 0.85y = 0.6(20)$$

$$0.4x + 0.85(20-x) = 12$$

$\underline{x = 11.12\,L}$

& $\underline{y = 8.88\,L}$ (2 marks)

#30 $2x + 6y = -3$ ①

$-6x - 18y = 5$ ②

Isolate x in ①

$\Longrightarrow x = -\frac{3}{2} - 3y$

substitute in ②

$$-6\left(-\frac{3}{2} - 3y\right) - 18y = 5$$

$$9 + 18y - 18y = 5$$

$$9 = 5$$

<u>no solution</u>. (2 marks)

#52

The distance travelled in air & water is assumed to be the same, it is what we're looking for: d.

we know distance = speed · time

<u>Air</u>

$d_{Air} = S_{Air} \cdot t_{Air}$

<u>water</u>

$d_{water} = S_{water} \cdot t_{water}$

equations

$d_{AIR} = d_{water}$ (1 mark)

$S_{AIR} = 330\,m/s$

$S_{water} = 1500\,m/s$

$t_{AIR} = t_{water} + 30$

Thus

$$330 \cdot (t_w + 30) = 1500 \cdot t_w$$

$$330t_w + 9900 = 1500\,t_w$$

$$t_w = 8.46 \text{ seconds}$$

so $d = (1500)\cdot(8.46) = \underline{12692.3\,m}$

Section 5.6

#20 $V_1 + V_2 + V_3 = 14000$ ①

$V_1 + 2V_2 = 13000$ ②

$3V_1 + 3V_2 + 2V_3 = 36000$ ③

Isolate V_1 in ② $V_1 = 13000 - 2V_2$

Substitute in ① $13000 - 2V_2 + V_2 + V_3 = 14000$

$-V_2 + V_3 = 1000$ ④

Substitute in ③ $3(13000 - 2V_2) + 3V_2 + 2V_3 = 36000$

$39000 - 6V_2 + 3V_2 + 2V_3 = 36000$

$-3V_2 + 2V_3 = -3000$ ⑤

We now have a new system with equations ④ & ⑤

isolate V_3 in ④ $V_3 = 1000 + V_2$

Substitute in ⑤ $-3V_2 + 2(1000 + V_2) = -3000$

$-3V_2 + 2000 + 2V_2 = -3000$

$-V_2 = -5000$

$V_2 = 5000$

$\Rightarrow$ Sub. $V_2 = 5000$

in $V_1 = 13000 - 2V_2$

$= 13000 - 2(5000)$

$= 3000$

(3 marks)

$\Rightarrow$ Sub. $V_1 = 3000$ & $V_2 = 5000$

in $V_3 = 1000 + V_2$

$= 1000 + 5000$

$= 6000$

soln. $(V_1, V_2, V_3) = (3000, 5000, 6000)$

Section 7.1

#4 $9x^2-12x+2=0$ is QUADRATIC

#6 $x(2x^2+5)=7+2x^2$

$2x^3+5x=7+2x^2$

$2x^3-2x^2+5x-7=0$ is NOT QUADRATIC

it has a variable to the third power

it is a cubic. (1 mark)

#10 by FACTORING

$B^2-400=0$

$(B+20)(B-20)=0$ $B=\pm 20$

#14 $x^2+x-6=0$

$(x+3)(x-2)=0$ $x=-3,2$

#16 $6z^2=6+5z$

$6z^2-5z-6=0$

$6z^2-9z+4z-6=0$ (2 marks)

$3z(2z-3)+2(2z-3)=0$

$(2z-3)(3z+2)=0$ $z=-\frac{2}{3}, \frac{3}{2}$

#28 $9t^2=9-t(43+t)$

$9t^2=9-43t-t^2$

$10t^2+43t-9=0$

$10t^2-2t+45t-9=0$

$2t(5t-1)+9(5t-1)=0$

$(5t-1)(2t+9)=0$ $t=\frac{1}{5}, -\frac{9}{2}$

#35 $(x+a)^2-b^2=0$

DIFFERENCE OF SQUARES $(x+a-b)(x+a+b)=0$ (1 mark)

$x=b-a, -a-b$

Section 7.2

Completing the square

14 $t^2+5t-6=0$

$t^2+5t = 6$

$(t+\frac{5}{2})^2 - \frac{25}{4} = 6$

$(t+\frac{5}{2})^2 = \frac{24}{4}+\frac{25}{4}$

$(t+\frac{5}{2})^2 = \frac{49}{4}$

$t+5/2 = \pm 7/2$

$t = -5/2 \pm 7/2$

$t = 1, -6$

18 $z^2+12 = 8z$

$z^2-8z = -12$

$(z-4)^2-16 = -12$

$(z-4)^2 = 4$

$z-4 = \pm 2$

$z = 6, 2$

24 $9v^2-6v-2=0$

$9v^2-6v = 2$

$v^2-\frac{2}{3}v = 2/9$

$(v-1/3)^2 - 1/9 = 2/9$

$(v-1/3)^2 = 1/3$

$v = \frac{+1}{-\sqrt{3}} + \frac{1}{3}$

$v = \frac{1}{3}+\frac{1}{\sqrt{3}}, \frac{1}{3}-\frac{1}{\sqrt{3}}$

(2 marks)

Section 7.3 Quadratic Formula

20 $40x^2-62x-63=0$

$x = \frac{62 \pm \sqrt{(-62)^2-4(40)(-63)}}{80}$

$= \frac{62 \pm \sqrt{3844+10080}}{80}$

$= \frac{62 \pm 118}{80}$

$= \frac{180}{80}, -\frac{56}{80}$ (2 marks)

$= 9/4, -7/10$

22 No solution, negative under the square root

26 $T = 37, 0$

Section 11.2

#20 $\dfrac{-7^{-1/2}}{6^{-1}\,7^{1/2}} = \boxed{\dfrac{-6}{7}}$

#22 $\dfrac{(-27)^{1/3}}{6} = \dfrac{\sqrt[3]{-27}}{6} = \dfrac{-3}{6} = \boxed{\dfrac{-1}{2}}$

#24 $\dfrac{-4^{-1/2}}{(-64)^{-2/3}} = \dfrac{-(-64)^{2/3}}{4^{1/2}} = -\dfrac{\left(\sqrt[3]{-64}\right)^2}{\sqrt{4}} = \dfrac{-(-4)^2}{2} = \dfrac{-16}{2} = \boxed{-8}$

(1 mark)

#28 $\dfrac{4^{-1}}{36^{-1/2}} - \dfrac{5^{-1/2}}{5^{1/2}} = \dfrac{36^{1/2}}{4^1} - \dfrac{1}{5} = \dfrac{6}{4} - \dfrac{1}{5} = \dfrac{3}{2} - \dfrac{1}{5}$

$= \dfrac{15}{10} - \dfrac{2}{10} = \boxed{\dfrac{13}{10}}$

(1 mark)

#40 $(8b^{-4}c^2)^{2/3}$

$= 8^{2/3} b^{-8/3} c^{4/3}$

$= \boxed{\dfrac{4c^{4/3}}{b^{8/3}}}$

#42 $(32c^5d^4)^{-2/5}$

$= \dfrac{1}{(32c^5d^4)^{2/5}}$

$= \dfrac{1}{32^{2/5} c^2 d^{8/5}}$

$= \dfrac{1}{\left[\sqrt[5]{32}\right]^2 c^2 d^{8/5}}$

$= \boxed{\dfrac{1}{4c^2 d^{8/5}}}$

#44 $\left(\dfrac{4a^{5/6}b^{-1/5}}{a^{2/3}b^2}\right)^{-1/2}$

$= \dfrac{4^{-1/2} a^{-5/12} b^{1/10}}{a^{-1/3} b^{-1}}$

$= \dfrac{4^{-1/2} a^{-5/12} b^{1/10}}{a^{-4/12} b^{-10/10}}$

$= \boxed{\dfrac{b^{11/10}}{2a^{1/12}}}$

(2 marks)

#49 $(T^{-1} + 2T^{-2})^{-1/2}$

$= \left(\dfrac{1}{T} + \dfrac{2}{T^2}\right)^{-1/2}$

$= \left(\dfrac{T}{T^2} + \dfrac{2}{T^2}\right)^{-1/2}$

$= \left(\dfrac{T+2}{T^2}\right)^{-1/2}$

$= \left(\dfrac{T^2}{T+2}\right)^{1/2}$ (2 marks)

$= \dfrac{T}{\sqrt{T+2}} \cdot \dfrac{\sqrt{T+2}}{\sqrt{T+2}}$

$= \boxed{\dfrac{T\sqrt{T+2}}{T+2}}$

Section 11.3

#6 $\sqrt{150} = \sqrt{25 \cdot 6} = \boxed{5\sqrt{6}}$

#10 $\sqrt{pq^2r^7} = \boxed{qr^3\sqrt{pr}}$

#12 $\sqrt{12ab^2} = \boxed{2b\sqrt{3a}}$ (1 mark)

#24 $\sqrt[7]{4}\sqrt[7]{64} = \sqrt[7]{2^2 \cdot 2^6}$

$= \boxed{2\sqrt{2}}$

#46 $\sqrt[5]{\sqrt[4]{9}}$

$= (9^{1/4})^{1/5} = 9^{1/20}$

$= \boxed{\sqrt[20]{9}}$ (1 mark)

$= (3^2)^{\frac{1}{20}} = 3^{1/10} = \boxed{\sqrt[10]{3}}$

#57 $\sqrt{xy^{-1} + x^{-1}y}$

$= \sqrt{\frac{x}{y} + \frac{y}{x}}$

$= \sqrt{\frac{x^2+y^2}{xy}}$ (1 mark)

$= \frac{\sqrt{x^2+y^2}}{\sqrt{xy}} \cdot \frac{\sqrt{xy}}{\sqrt{xy}}$

$= \boxed{\frac{\sqrt{xy(x^2+y^2)}}{xy}}$

#48 $\sqrt{b^4\sqrt{a}} = \sqrt{b^4 a^{1/2}}$ (1 mark)

$= (b^4 a^{1/2})^{1/2}$

$= b^2 a^{1/4}$

$= \boxed{b^2\sqrt[4]{a}}$

#55 $\sqrt{\frac{5}{4} - \frac{1}{8}} = \sqrt{\frac{10}{8} - \frac{1}{8}}$

$= \sqrt{9/8}$

$= \frac{3}{\sqrt{8}}$

$= \frac{3}{2\sqrt{\ }}$

$= \frac{3 \cdot \sqrt{\ }}{2\sqrt{\ } \cdot \sqrt{\ }}$

$= \frac{3\sqrt{\ }}{\ }$

$=$

(1 mark)

Section 11.5

#28 $(2\sqrt{mn} + 3\sqrt{n})^2$

$= 4mn + 12\sqrt{mn}\sqrt{n} + 9n$

$= \boxed{4mn + 12n\sqrt{m} + 9n}$

#30 $\sqrt[5]{16}\sqrt[3]{8} = 2^{4/5} \cdot 2 = \boxed{2^{9/5}}$ (1 mark)

#32 $\dfrac{6\sqrt[4]{25}}{5 - 2\sqrt{5}}$

$= \dfrac{6(5^{2/2})^{1/4}}{5 - 2\sqrt{5}}$

$= \dfrac{6 \cdot 5^{1/2}}{5 - 2\sqrt{5}}$

$= \dfrac{6\sqrt{5}}{5 - 2\sqrt{5}} \cdot \dfrac{(5 + 2\sqrt{5})}{(5 + 2\sqrt{5})}$

$= \dfrac{30\sqrt{5} + 60}{25 - 20} = \dfrac{30\sqrt{5} + 60}{5}$

$= 6\sqrt{5} + 12$

$= \boxed{6(\sqrt{5} + 2)}$

(2 marks)

#38 $\dfrac{\sqrt{2c} + 3d}{\sqrt{2c} - d}$

$= \dfrac{\sqrt{2c} + 3d}{\sqrt{2c} - d} \cdot \dfrac{(\sqrt{2c} + d)}{(\sqrt{2c} + d)}$

$= \boxed{\dfrac{2c + 4d\sqrt{2c} + 3d^2}{2c - d^2}}$

#44 $\dfrac{\sqrt{1+a}}{a - \sqrt{1-a}} \cdot \dfrac{a + \sqrt{1-a}}{a + \sqrt{1-a}} = \dfrac{a\sqrt{1+a} + \sqrt{1-a^2}}{a^2 - (1-a)}$

$= \boxed{\dfrac{a\sqrt{1+a} + \sqrt{1-a^2}}{a^2 + a - 1}}$