

ASSIGNMENT # 4

201-943-DW S01

Fall 2014

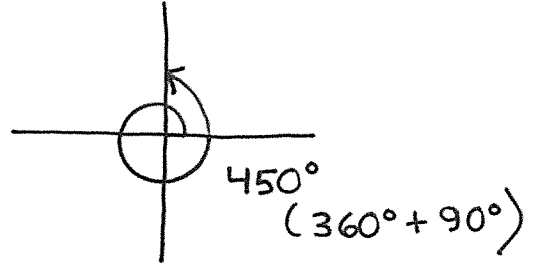
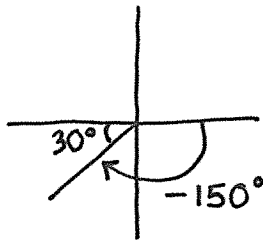
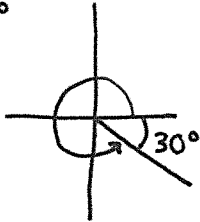
Instructor E. RICHEN

Dec 5th 2014

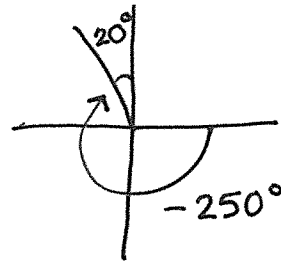
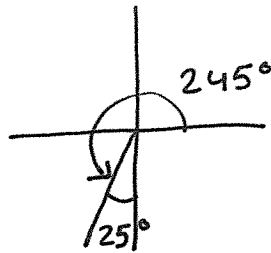
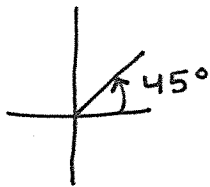
Section 4.1

(solutions to odd numbers are in the textbook)

6. 330°



8. 45°



30. 715.80°

$= 715^\circ 48'$

32. 142.87°

$= 142^\circ 52.2'$

$= 142^\circ 52' 12''$

34. $517^\circ 39'$

$= 517.65^\circ$

36. $-4^\circ 47'$

$= -4.78^\circ$

5 marks

Section 8.3

6. $12^\circ = \frac{1}{15} \pi$ $225^\circ = \frac{5}{4} \pi$

10. $5^\circ = \frac{1}{36} \pi$ $300^\circ = \frac{5}{3} \pi$

12. $66^\circ = \frac{11}{30} \pi$ $540^\circ = 3\pi$

5 MARKS

22. $54.3^\circ = 0.948$

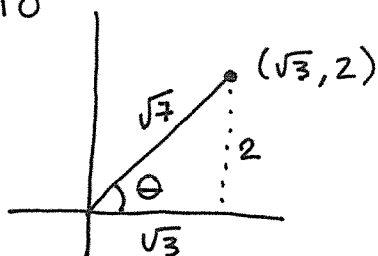
30 13.75°

24 $104^\circ = 1.81$

32 $1.703 = 97.57^\circ$

Section 4.2

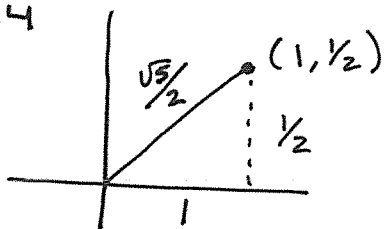
10



$$r^2 = \sqrt{3}^2 + 2^2$$
$$r = \sqrt{7}$$

$$\sin \theta = 2/\sqrt{7} \quad \cos \theta = \sqrt{3}/\sqrt{7} \quad \tan \theta = 2/\sqrt{3}$$
$$\csc \theta = \sqrt{7}/2 \quad \sec \theta = \sqrt{7}/\sqrt{3} \quad \cot \theta = \sqrt{3}/2$$

14



$$r^2 = 1^2 + \frac{1}{2}^2$$
$$= 1 + \frac{1}{4} = \frac{5}{4}$$
$$r = \sqrt{5/4} = \frac{\sqrt{5}}{2}$$

$$\sin \theta = \frac{1/2}{\sqrt{5}/2} = \frac{1}{\sqrt{5}} \quad \csc \theta = \sqrt{5}$$

$$\cos \theta = \frac{1}{\sqrt{5}/2} = \frac{2}{\sqrt{5}} \quad \sec \theta = \frac{\sqrt{5}}{2}$$

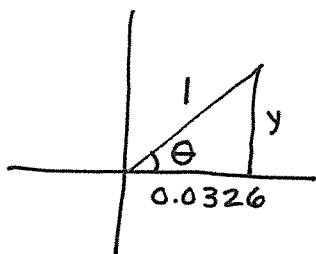
$$\tan \theta = \frac{1/2}{1} = \frac{1}{2} \quad \cot \theta = 2$$

QUESTIONS 10, 14

3 MARKS

22 $\cos \theta = 0.0326$ Find $\sin \theta$ & $\tan \theta$

$$\cos \theta = \frac{x}{r} = \frac{0.0326}{1}$$



$$1^2 = 0.0326^2 + y^2$$

$$y^2 = 0.9989$$

$$y = \pm 0.9995$$

$$\sin \theta = \pm \frac{0.9995}{1} = \pm 0.9995$$

$$\tan \theta = \pm \frac{0.9995}{0.0326} = \pm 30.66$$

24

$$\csc \theta = 1.2$$

Find $\sec \theta$ & $\cos \theta$

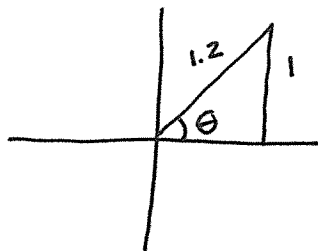
Page 3.

$$\frac{1}{\sin \theta} = 1.2$$

$$\sin \theta = \frac{1}{1.2} = \frac{y}{r}$$

$$\cos \theta = \pm \frac{0.663}{1.2} = \pm 0.55$$

$$\sec \theta = \pm \frac{1.2}{0.663} = \pm 1.81$$



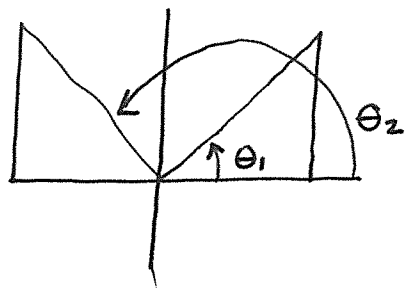
$$x^2 + 1^2 = 1.2^2$$

$$x = \pm 0.663$$

QUESTIONS 22 & 24

2 MARKSSection 8.2

30 $\sin \theta = 0.6374$



$$\sin^{-1}(0.6374) = 39.6^\circ$$

$$\text{So } \theta_1 = 39.6^\circ$$

$$\& \theta_2 = 180^\circ - 39.6^\circ = 140.4^\circ$$

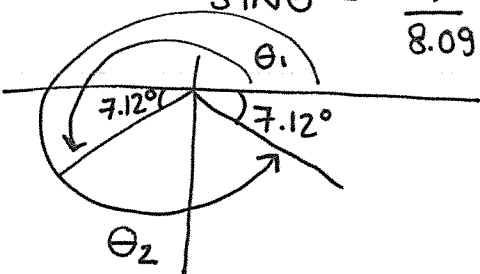
SECTION 8.2

10 MARKS

32 $\csc \theta = -8.09$

$$\sin \theta = -\frac{1}{8.09} = -0.124$$

$$\sin^{-1}(-0.124) = -7.12^\circ$$



$$\theta_1 = 180^\circ + 7.12^\circ = 187.12^\circ$$

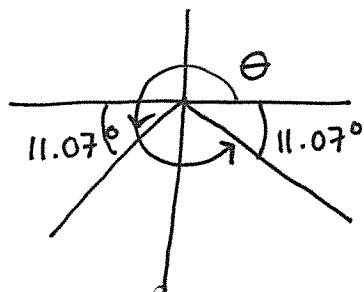
$$\theta_2 = 360^\circ - 7.12^\circ = 352.88^\circ$$

36 $\sin \theta = -0.192$ $\tan \theta < 0$

$$\sin^{-1}(-0.192) = -11.07^\circ$$

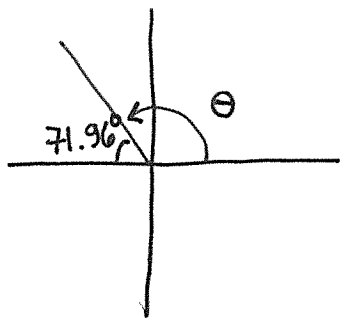
$$\theta = 360^\circ - 11.07^\circ$$

$$= 348.93^\circ$$



ONLY ONE QUADRANT
where $\sin \theta$ & $\tan \theta$
are both negative

40 $\cot \theta = -0.3256$, $\csc \theta > 0$



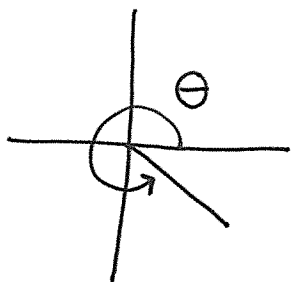
$$\tan \theta = \frac{1}{-0.3256}$$

$$\tan^{-1}\left(\frac{-1}{0.3256}\right) = -71.96^\circ$$

$$\begin{aligned} \theta &= 180^\circ - 71.96^\circ \\ &= 108.04^\circ \end{aligned}$$

ONE QUADRANT
WHERE TAN IS NEGATIVE
& SIN POSITIVE

42 $\cos \theta = 0.422$ $\tan \theta < 0$ Find $\sin \theta$.



$\cos \theta > 0$
 $\tan \theta < 0$

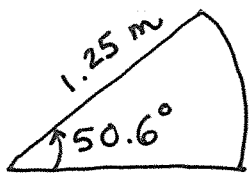
$$\cos^{-1}(0.422) = 65.04^\circ$$

$$\begin{aligned} \theta &= 360 - 65.04^\circ \\ &= 294.96^\circ \end{aligned}$$

$$\begin{aligned} \sin \theta &= \sin 294.96^\circ \\ &= -0.907 \end{aligned}$$

Section 8.4

20



$$A = \frac{1}{2} \theta r^2$$

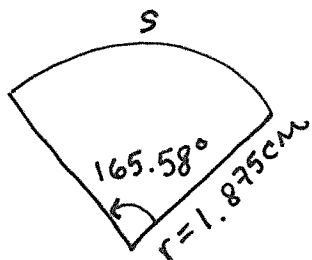
θ in rads

$$\begin{aligned} A &= \frac{1}{2} (0.883) (1.25)^2 \\ &= \underline{0.69 \text{ m}^2} \end{aligned}$$

$$50.6^\circ = 0.883$$

1 mark

22



$$\theta \text{ in rad} = 2.89$$

$$\theta = \frac{S}{r}$$

$$2.89 = \frac{S}{1.875}$$

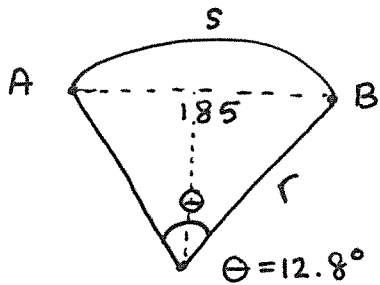
$$S = 5.42 \text{ cm}$$

$$\begin{aligned} \text{Perimeter} &= S + 2r \\ &= 5.42 + 2(1.875) \\ &= \underline{9.17 \text{ cm}} \end{aligned}$$

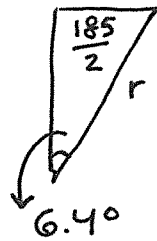
1 mark

35

page 2.



Find r:



$$\sin 6.4^\circ = \frac{92.5}{r}$$

$$r = 829.8 \text{ km}$$

Length of arc s:

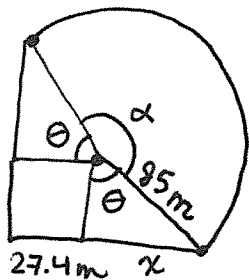
$$\theta = \frac{s}{r} \rightarrow 0.223 = \frac{s}{829.8}$$

$$s = 185.38 \text{ km}$$

2 marks

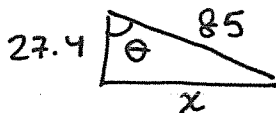
The pilot flies an extra
0.38 km.

54



The field has a sector of a circle,
2 triangles & one square:

Triangles:



$$\cos \theta = \frac{27.4}{85}$$

$$\theta = \cos^{-1}\left(\frac{27.4}{85}\right) = 71.19^\circ$$

$$x^2 + 27.4^2 = 85^2$$

$$x = 80.46 \text{ m}$$

$$\begin{aligned} \text{Area of triangle} &= \frac{1}{2}(27.4)(80.46) \\ &= 1,102.3 \text{ m}^2 \end{aligned}$$

BONUS

3 marks

Area of sector



$$\begin{aligned} \alpha &= 360^\circ - 90^\circ - 71.19^\circ(2) \\ &= 127.62^\circ \\ &= 2.227 \end{aligned}$$

$$\begin{aligned} A &= \frac{1}{2}\theta r^2 = \frac{1}{2}(2.227)85^2 \\ &= 8046.4 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of square} &= (27.4)^2 \\ &= 750.76 \text{ m}^2 \end{aligned}$$

Total area of Field =

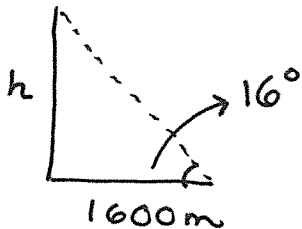
$$\square + 2 \triangle + \text{sector}$$

$$= 750.76 + 2(1,102.3) + 8046.4$$

$$= \underline{11,002 \text{ m}^2}$$

Section 4.5

10

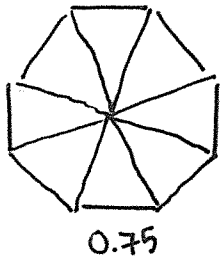


$$\tan 16^\circ = \frac{h}{1600}$$

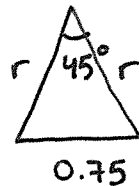
$$h = \underline{458.79 \text{ m}}$$

1 mark

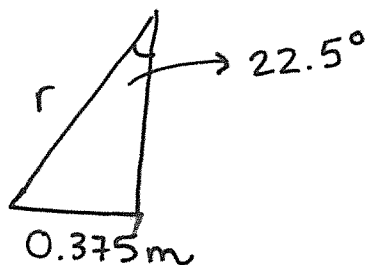
11



one eighth



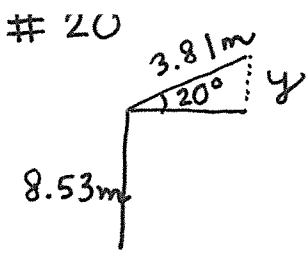
divide triangle in two so that we have a right angle triangle:



$$\sin 22.5^\circ = \frac{0.375}{r}$$

$$r = 0.9799$$

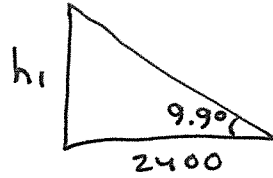
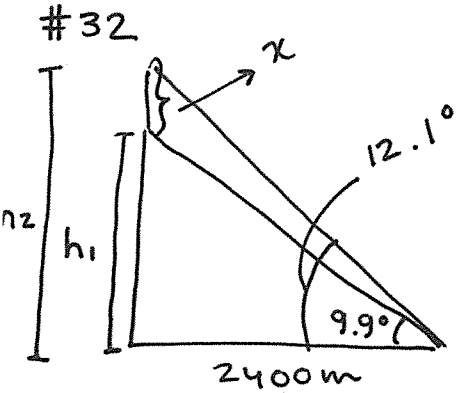
so the longest distance across the table is 1.96 m.



$$\sin 20^\circ = \frac{y}{3.81} \rightarrow y = 1.303$$

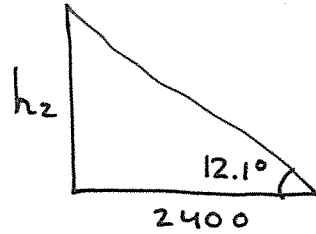
$$\text{height} = 8.53 + 1.303 \\ = \underline{9.833 \text{ m}}$$

2 marks



$$\tan 9.9 = \frac{h_1}{2400}$$

$$h_1 = 418.87 \text{ m}$$



$$\tan 12.1 = \frac{h_2}{2400}$$

$$h_2 = 514.515 \text{ m}$$

$$\text{Height of ANTENNA} = h_2 - h_1 \\ = \underline{95.648 \text{ m}}$$

3 marks

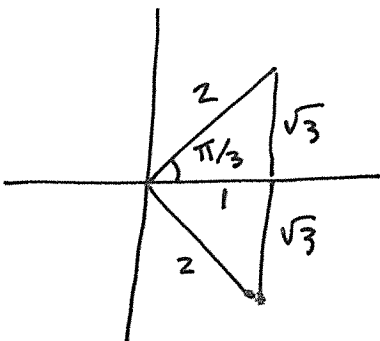
Section 20.5

(8) $4 \cos^2 x - 1 = 0$

$$\cos^2 x = \frac{1}{4}$$

$$\cos x = \pm \sqrt{\frac{1}{4}} = \pm \frac{1}{2}$$

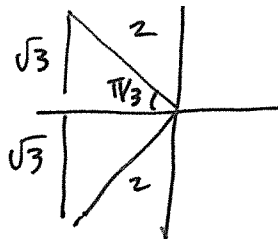
$$\cos x = \frac{1}{2}$$



$$\theta_1 = \pi/3$$

$$\theta_2 = 2\pi - \pi/3 = 5\pi/3$$

$$\cos x = -\frac{1}{2}$$



$$\theta_3 = \pi - \pi/3 = 2\pi/3$$

$$\theta_4 = \pi + \pi/3 = 4\pi/3$$

Four solutions $\pi/3, 2\pi/3, 4\pi/3, 5\pi/3$

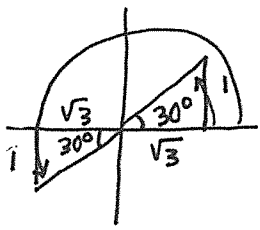
10 $3 \tan^2 x - 1 = 0$

page 0.

$\tan^2 x = \frac{1}{3}$

$\tan x = \pm \sqrt{\frac{1}{3}} = \pm \frac{1}{\sqrt{3}}$

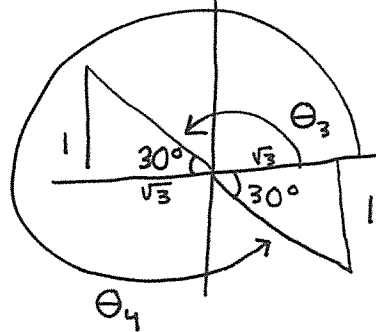
$\tan x = \frac{1}{\sqrt{3}}$



$\theta_1 = 30^\circ = \pi/6$

$\theta_2 = 180^\circ + 30^\circ = 210^\circ = 7\pi/6$

$\tan x = -\frac{1}{\sqrt{3}}$



$\theta_3 = 150^\circ = \frac{5\pi}{6}$

$\theta_4 = \frac{11\pi}{6}$

Four solutions

$\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

16

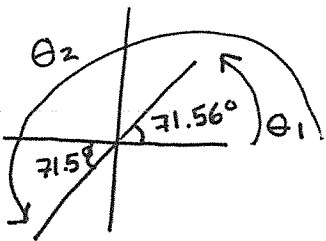
$\tan^2 x + 6 = 5 \tan x$

$\tan^2 x - 5 \tan x + 6 = 0$

$(\tan x - 3)(\tan x - 2) = 0$

$\tan x = 3 \quad \tan x = 2$

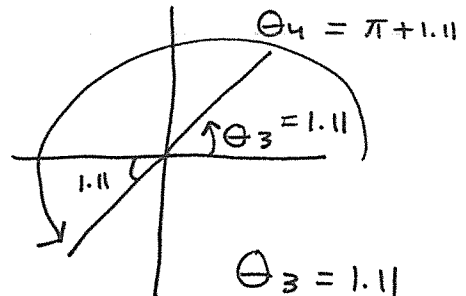
$\tan^{-1}(3) = 71.56^\circ$



$\theta_1 = 71.56^\circ = 1.25$

$\theta_2 = 4.39$

$\tan^{-1}(2) = 1.1 \text{ rad}$



$\theta_3 = 1.1$

$\theta_4 = 4.25$

Solutions: 1.11, 1.25, 4.25, 4.39

8, 16

6 marks

$$\# 18 \quad a. \sqrt{(-15)^2} \quad b. (\sqrt{-15})^2$$

$$= \underline{15} \quad = \underline{-15}$$

$$\# 20 \quad a. \sqrt{-9} \sqrt{-16} \quad b. \sqrt{(-9)(-16)}$$

$$= (3j)(4j) \quad = \sqrt{144}$$

$$= 12j^2 \quad = \underline{12}$$

$$= \underline{-12}$$

$$\# 22 \quad -\sqrt{\left(\frac{-4}{7}\right)\left(\frac{-49}{16}\right)}$$

$$= -\sqrt{\frac{7}{4}}$$

$$= \underline{\frac{-\sqrt{7}}{2}}$$

$$\# 26 \quad a. -j^{21}$$

$$= -j^{20} j$$

$$= \underline{-j}$$

$$b. (-j)^{21}$$

$$= (-1)^{21} j^{21}$$

$$= -1 j^{20} j$$

$$= \underline{-j}$$

$$\# 30 \quad 3j^{48} + j^{200} = 3(j^4)^{12} + (j^4)^{50}$$

$$= 3(1) + (1)$$

$$= \underline{4}$$

4 marks

$$\# 52 \quad 2x + 3jy = -6 + 12j$$

$$\text{Real: } 2x = -6 \rightarrow \underline{x = -3}$$

$$\text{Imag: } 3y = 12 \rightarrow \underline{y = 4}$$

$$\# 54 \quad 2x - 6xj^3 - 3j^2 = yj - y + 7j^5$$

$$2x - 6x(-j) - 3(-1) = yj - y + 7j$$

$$2x + 6xj + 3 = yj - y + 7j$$

2 marks

$$\text{Real: } 2x + 3 = -y \quad \textcircled{1} \quad \rightarrow \quad y = -2x - 3$$

$$\text{Imag: } 6x = y + 7 \quad \textcircled{2}$$

$$\text{sub in } \textcircled{2} \quad 6x = -2x - 3 + 7$$

$$8x = 4$$

$$\underline{x = \frac{1}{2}}$$

$$y = -2\left(\frac{1}{2}\right) - 3 = \underline{-4}$$

Section 12.2

$$\begin{aligned}
 \# 18 \quad & \sqrt{-6} \sqrt{-12} \sqrt{30} \\
 & = \sqrt{6}j \sqrt{12}j \sqrt{30} \\
 & = \sqrt{3}\sqrt{2}j \sqrt{3}\sqrt{4}j \sqrt{2}\sqrt{3}\sqrt{5} \\
 & = \sqrt{3}\sqrt{3}\sqrt{2}\sqrt{2}\sqrt{4}\sqrt{3}\sqrt{5}j^2 \\
 & = 3 \cdot 2 \cdot 2 \sqrt{15} (-1) \\
 & = \underline{-12\sqrt{15}}
 \end{aligned}$$

$$\begin{aligned}
 \# 22 \quad & j^2 \sqrt{-7} - \sqrt{-28} + 8 \\
 & = -1 \sqrt{-1}\sqrt{7} - \sqrt{-1}\sqrt{28} + 8 \\
 & = -\sqrt{7}j - 2\sqrt{7}j + 8 \\
 & = \underline{8 - 3\sqrt{7}j}
 \end{aligned}$$

$$\begin{aligned}
 \# 28 \quad & \frac{0.25}{3 - \sqrt{-1}} \\
 & = \frac{(0.25)(3+j)}{(3-j)(3+j)} \\
 & = \frac{0.75 + 0.25j}{9 + 1} \\
 & = \frac{0.75}{10} + \frac{0.25j}{10} \\
 & = \underline{0.075 + 0.025j}
 \end{aligned}$$

$$\begin{aligned}
 \# 30 \quad & \frac{6+5j}{3-4j} \\
 & = \frac{(6+5j)(3+4j)}{(3-4j)(3+4j)} \\
 & = \frac{18 + 24j + 15j + 20j^2}{9 - 16j^2} \\
 & = \frac{-2 + 39j}{25} = \underline{-\frac{2}{25} + \frac{39}{25}j}
 \end{aligned}$$

2 marks

$$\# 42 \quad (1+j)^{-3} (2-j)^{-2}$$

$$= \frac{1}{(1+j)^3} \cdot \frac{1}{(2-j)^2}$$

$$= \frac{1}{(-2+2j)} \cdot \frac{1}{(3-4j)}$$

$$= \frac{1}{-6 + 6j + 8j - 8j^2}$$

$$= \frac{1}{2+14j} = \frac{1}{(2+14j)(2-14j)}$$

$$= \frac{2-14j}{200} = \underline{\frac{1}{100} - \frac{7}{100}j}$$

$$\Rightarrow (1+j)^3 = (1+j)(1+j)(1+j)$$

$$= (1+2j+j^2)(1+j)$$

$$= (2j)(1+j)$$

$$= 2j + 2j^2 = -2 + 2j$$

$$= 2j + 2j^2 = -2 + 2j$$

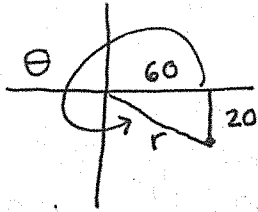
3 marks

$$\Rightarrow (2-j)^2 = (2-j)(2-j)$$

$$= 4 - 4j + j^2$$

$$= 3 - 4j$$

#28 $60 - 20j$



$$r^2 = 60^2 + 20^2$$

$$r = \sqrt{4000}$$

$$= 20\sqrt{10}$$

$$\tan^{-1}\left(\frac{20}{60}\right) = 18.43^\circ$$

$$\theta = 360^\circ - 18.43^\circ$$

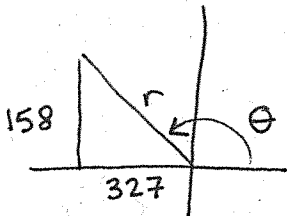
$$= 341.57^\circ$$

Polar: $20\sqrt{10} / 341.57^\circ$

exponential: $20\sqrt{10} e^{5.96j}$

2 marks

#30 $-327 + 158j$



$$r^2 = 327^2 + 158^2$$

$$r = 363.2$$

$$\tan^{-1}\left(\frac{158}{327}\right) = 25.79^\circ$$

$$\theta = 180^\circ - 25.79^\circ$$

$$= 154.21^\circ$$

$$= 2.69 \text{ rads}$$

polar: $363.2 / 154.21^\circ$

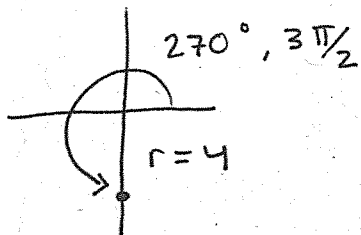
exp: $363.2 e^{2.69j}$

#32 $-4j$

$$= -4j$$

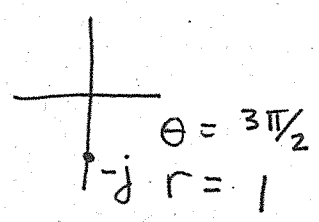
polar $4 / 270^\circ$

exp $4 e^{3\pi/2j}$



2 MARKS

67 $x^4 + j = 0$
 $x = \sqrt[4]{-j}$



→ 4 EXPONENTIAL FORMS FOR $-j$

- ① $e^{3\pi/2j}$
- ② $e^{7\pi/2j}$
- ③ $e^{11\pi/2j}$
- ④ $e^{15\pi/2j}$

Applying 4th root:

① $(e^{3\pi/2j})^{1/4} = e^{3\pi/8j} = \cos 3\pi/8 + j \sin 3\pi/8 = 0.38 + 0.92j$

② $(e^{7\pi/2j})^{1/4} = e^{7\pi/8j} = \cos 7\pi/8 + j \sin 7\pi/8 = -0.92 + 0.38j$

③ $(e^{11\pi/2j})^{1/4} = e^{11\pi/8j} = \cos 11\pi/8 + j \sin 11\pi/8 = -0.38 - 0.92j$

④ $(e^{15\pi/2j})^{1/4} = e^{15\pi/8j} = -0.38 - 0.92j$

75 $x^2 + 3jk - 2 = 0$

2 MARKS

$$x = \frac{-3j \pm \sqrt{(3j)^2 - 4(1)(-2)}}{2}$$

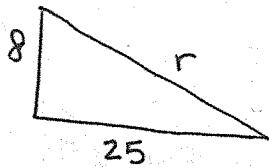
$$= \frac{-3j \pm \sqrt{-1}}{2}$$

$$= \frac{-3j \pm j}{2}$$

1 mark

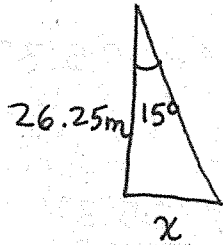
Two solutions are $-2j$ & $-j$

Bonus Chapter 4 Review # 94



$$r^2 = 8^2 + 25^2$$

$$r = 26.25 \text{ m}$$



$$\tan 15^\circ = \frac{x}{26.25}$$

$$x = 7.03 \text{ m}$$

$$\text{width} = 2x = 14.06 \text{ m}$$