NAME: solutions

STUDENT NUMBER: ____________________

TEST 1

Dawson College
Applied Math (201-943-DW-S01)
Date: SEPTEMBER 26th 2014
Instructor: Emilie Richer

This test is scored out of 100 marks

Question 1. *(10 marks)*
How much sand must be added to 200kg of a cement mixture that is 27% sand in order to obtain a mixture that is 30% sand?

Mixture	+	Sand	=	Desired Mixture
27% sand		100% sand		30% sand
200 Kg		x Kg		$200 + x$ Kg

$$200(0.27) + x(1) = (200 + x)(0.3)$$

$$54 + x = 60 + 0.3x$$

$$0.7x = 6$$

$$x = 6/0.7 = 8.57 \text{ Kg}$$

Approximately 8.57 Kg of sand must be added to the cement mixture.

Question 2. *(2 marks each = 8 marks)*

(a) $-\sqrt{36} = -6$

(b) $(4^2 + 3^2)^{-1/2} = (16+9)^{-1/2} = 25^{-1/2} = \dfrac{1}{25^{1/2}} = \dfrac{1}{\sqrt{25}} = \dfrac{1}{5}$

(c) $\sqrt[5]{-32} = -2$

(d) $\sqrt[3]{-64x^3} = -4x$

Question 3. *(10 marks)*
Perform the indicated operations.

(a) $ax(cx^2)(x+y^3)$

$= acx^3(x+y^3)$

$= \boxed{acx^4 + acx^3y^3}$

(b) $9v-[6-(v-4)+4v]$

$= 9v - [6-v+4+4v]$

$= 9v - [10+3v]$

$= \boxed{6v-10}$

Question 4. *(5 marks)*
Perform the indicated division:

$\frac{8x^3+4x^2-2x+3}{x-1}$

$= \boxed{8x^2+12x+10 + \frac{13}{x-1}}$

$$
\begin{array}{r}
8x^2 + 12x + 10 \\
x-1 \overline{\smash{)}\, 8x^3+4x^2-2x+3} \\
-(8x^3-8x^2) \\
\hline
12x^2-2x+3 \\
-(12x^2-12x) \\
\hline
10x+3 \\
-(10x-10) \\
\hline
13
\end{array}
$$

Question 5. *(10 marks)*
Solve the following equations:
(a) $5-4(1-7t)=2t-5$

$5-4+28t=2t-5$
$26t=-6$
$t=\frac{-6}{26}=\boxed{\frac{-3}{13}}$

(b) $\frac{2x-3(-x+4)}{6}=5$

$2x-3(-x+4)=30$
$2x+3x-12=30$
$5x=42$
$\boxed{x=42/5}$

Question 6. *(5 marks)*
Given the following relationship between variables s and t:
$t^2=s-3$
State whether s is a function of t or whether t is a function of s. Explain your answer, using the definition of a function.

S is a function of t, $s=t^2+3$;
because for every value of t, there is a single value of s that corresponds.

However t is not a function of s, because for a choice of s there is more than one possible corresponding value of t:

Example $S=12$
$t^2=12-3=9$
t is possible 3 or -3

Question 7. *(2 marks each = 10 marks)*
Given the functions

$f(x) = x^3 - 2$

$g(t) = \sqrt{t-1}$

$h(s) = \frac{1}{1-s}$

Find the following values.

(a) $f(0)$ $= 0^3 - 2 = \boxed{-2}$

(b) $g(x+3)$ $= \sqrt{x+3-1} = \boxed{\sqrt{x+2}}$

(c) $h(\sqrt{5})$ $= \boxed{\frac{1}{1-\sqrt{5}}}$

(d) $f(g(5))$ $= f(\sqrt{5-1}) = f(\sqrt{4}) = f(2) = 2^3 - 2 = \boxed{6}$

(e) $2h(4) - [f(1)]^2$ $= 2\left(\frac{1}{1-4}\right) - (1^3 - 2)^2 = \frac{2}{-3} - (-1)^2$

$= -2/3 - 1 = \boxed{-5/3}$

Question 8. *(5 marks)*
State the domain and range of the function $H(h) = 2h + \sqrt{h} + 4$. Explain your answer.

Domain: $\{h \in \mathbb{R} \mid h \geq 0\}$ b/c no negatives under the square root

Range: $H(h) \geq 4$

Because h must be ≥ 0

& $\sqrt{h}$ must be ≥ 0

so $2h + \sqrt{h}$ must be ≥ 0

& therefore $H(h) = 2h + \sqrt{h} + 4 \geq 4$

Question 9. *(5 marks)*
Solve the following equation:
$\frac{7}{y} = \frac{3}{y-4} + \frac{7}{2y^2-8y}$

$$\frac{7}{y} = \frac{3}{y-4} + \frac{7}{2y(y-4)}$$

so $y \neq 0$ & $y \neq 4$
common denominator:
$2y(y-4)$

$$\frac{14(y-4)}{2y(y-4)} = \frac{6y}{2y(y-4)} + \frac{7}{2y(y-4)}$$

$$\cancel{28}y - 56 = 6y + 7$$

$$8y = 63$$

$$\boxed{y = 63/8}$$

Question 10. *(5 marks)*
Solve for Q in the equation $f = \frac{PQ}{P+Q}$

$$f = \frac{PQ}{P+Q}$$

$$f(P+Q) = PQ$$

$$fP + fQ = PQ$$

$$fQ - PQ = fP$$

$$Q(-f+P) = fP$$

$$\boxed{Q = \frac{fP}{-f+P}}$$

Question 11. *(3 marks each = 15 marks)*
Factor each expression completely.
(a) $4a^2x^2 + 26a^2x + 36a^2$

$= 2a^2(2x^2 + 13x + 18)$
$= 2a^2(2x^2 + 4x + 9x + 18)$
$= 2a^2[2x(x+2) + 9(x+2)]$
$= \boxed{2a^2(x+2)(2x+9)}$

(b) $8x^3 + 27$

$= (2x)^3 + 3^3$ SUM OF CUBES
$= \boxed{(2x+3)(4x^2 - 6x + 9)}$

(c) $x^2 + x - 56$
$= \boxed{(x+8)(x-7)}$

(d) $16(x+2)^2 - t^4$ difference OF SQUARES
$= \boxed{(4(x+2) - t^2)(4(x+2) + t^2)}$

(e) $8x^2 - 8x - 70$
$= 2(4x^2 - 4x - 35)$ $N \cdot M = -140$
$N + M = -4$
$= 2(4x^2 - 14x + 10x - 35)$ $N, M = -14, 10$
$= 2(2x(2x-7) + 5(2x-7))$
$= \boxed{2(2x-7)(2x+5)}$

Question 12. *(3 marks each = 12 marks)*
Simplify the given expressions. Express results with positive exponents only.
(a) $-2(mn^{-5})(8m^{-3}n^{4})$

$= -2 \cdot 8 \cdot m \cdot m^{-3} \cdot n^{-5} \cdot n^{4}$

$= -16\, m^{-2} n^{-1} = \boxed{\dfrac{-16}{m^{2}n}}$

(b) $\frac{-35x^{-1}y(x^{2}y)}{5xy^{-1}}$

$= \dfrac{-7\, y(x^{2}y))y}{x \cdot x} = \dfrac{-7\, x^{2}y^{3}}{x^{2}} = \boxed{-7y^{3}}$

(c) $\frac{-2a^{-2}b^{4}(ab)^{-1}}{(2^{2}a)^{3}b^{-2}}$ $= \dfrac{-2a^{-2}b^{4}a^{-1}b^{-1}}{2^{6}a^{3}\, b^{-2}}$

$= \dfrac{-2\, a^{-3}b^{3}}{2^{6}\, a^{3}\, b^{-2}} = \boxed{\dfrac{-b^{5}}{2^{5}a^{6}}}$

(d) $\left(\frac{v^{-1}}{2t}\right)^{-2}\left(\frac{t^{2}}{v^{-2}}\right)^{-3}$

$= \dfrac{v^{2}}{2^{-2}t^{-2}} \cdot \dfrac{t^{-6}}{v^{6}} = \boxed{\dfrac{2^{2}}{t^{4}v^{4}}}$