NAME: ______________________

STUDENT NUMBER: SOLUTIONS

TEST 2

Dawson College
Applied Math (201-943-DW-S01)
Date: OCTOBER 31st 2014
Instructor: Emilie Richer

This test is scored out of 50 marks

Question 1. *(2 marks each = 8 marks)*
Perform the indicated operations and express the result in simplest radical form.

(a) $\sqrt[4]{64r\ s^4t^5}$

$= \sqrt[4]{2^6 r\ s^4 t^5} = \sqrt[4]{2^4 \cdot 2^2 r^2 s^4 t^4 \cdot t}$

$= 2st\sqrt[4]{2^2 r^2 t} = 2st\,(2^2r^2)^{1/4}\ t^{1/4}$

$= \boxed{2st\sqrt{2r}\ \sqrt[4]{t}}$

(b) $\sqrt[5]{\sqrt[4]{9}}$

$= \sqrt[5]{(9)^{1/4}}$

$= ((9)^{1/4})^{1/5} = 9^{1/20} = (3^2)^{1/20} = 3^{1/10} = \boxed{\sqrt[10]{3}}$

(c) $\frac{\sqrt{2}}{\sqrt{3}-4\sqrt{2}}$

$= \frac{\sqrt{2}}{\sqrt{3}-4\sqrt{2}} \cdot \frac{(\sqrt{3}+4\sqrt{2})}{(\sqrt{3}+4\sqrt{2})}$

$= \frac{\sqrt{2}\sqrt{3}+4\sqrt{2}\sqrt{2}}{3-16\cdot 2} = \boxed{\frac{\sqrt{6}+8}{-29}}$

(d) $\left(2s^{-1}+t\right)^{-2}$

$= \frac{1}{(2s^{-1}+t)^2}$

$= \frac{1}{\left(\frac{2}{s}+t\right)^2} = \frac{1}{\left(\frac{2+ts}{s}\right)^2} = \boxed{\frac{s^2}{(2+ts)^2}}$

Question 2. *(3 marks)*
Solve the following quadratic equation by **completing the square**.

$2x^2+16x-9=0$

$$2(x^2+8x-9/2)=0$$
$$x^2+8x = 9/2$$
$$(x+4)^2-16 = 9/2$$
$$(x+4)^2 = 9/2+16$$
$$(x+4)^2 = \frac{41}{2}$$
$$x+4 = \pm\sqrt{\frac{41}{2}}$$
$$\boxed{x = -4 \pm \sqrt{\frac{41}{2}}}$$

Question 3. *(4 marks)*
Express the following as a single logarithm

$2\log_3 x+\log_3 25-\log_3(2x)+\frac{\log(4x)}{\log 3}$

$$= \log_3 x^2 + \log_3 25 - \log_3(2x) + \log_3(4x)$$
$$= \log_3\left(\frac{x^2\cdot 25\cdot 4x}{2x}\right) = \log_3\left(x^2\cdot\frac{100}{2}\right)$$
$$= \boxed{\log_3(50x^2)}$$

Question 4. *(3 marks)*
Given the function

$$f(x)=\begin{cases} 3x-4 & \text{for } x>1 \\ -2x\sqrt{1-x} & \text{for } x\leq 1 \end{cases}$$

find $f(1)$, $f(2)$ and $f(-3)$.

$$f(1) = -2(1)\sqrt{1-1} = \boxed{0}$$
$$f(2) = 3(2)-4 = \boxed{2}$$
$$f(-3) = -2(-3)\sqrt{1-(-3)}$$
$$= 6\sqrt{4} = \boxed{12}$$

Question 5. *(4 marks each = 8 marks)*
Solve the following equations:

(a) $3^{3x+1} = 8$

$$\log_3 3^{3x+1} = \log_3 8$$
$$3x+1 = \log_3 8$$
$$3x = \log_3 8 - 1$$
$$x = \boxed{\frac{\log_3 8 - 1}{3}} \approx 0.298$$

(b) $2 \times 5^{x-1} = e^{-x}$

$$\ln(2 \cdot 5^{x-1}) = \ln e^{-x}$$
$$\ln 2 + \ln 5^{x-1} = -x$$
$$\ln 2 + (x-1)\ln 5 = -x$$
$$\ln 2 + x\ln 5 - \ln 5 = -x$$
$$x\ln 5 + x = \ln 5 - \ln 2$$
$$x(\ln 5 + 1) = \ln 5 - \ln 2$$
$$x = \frac{\ln 5 - \ln 2}{\ln 5 + 1} = \frac{\ln 5 - \ln 2}{\ln 5 + \ln e} = \boxed{\frac{\ln(5/2)}{\ln(5e)}}$$
$$\approx 0.351$$

Question 6. *(4 marks each = 12 marks)*
Solve the following equations:

(a) $\log_x 64 = 3$

$x^3 = 64$

$x = \sqrt[3]{64} = \boxed{4}$

(b) $\ln(x-2) + \ln(2x+1) = 2\ln x$

$\ln[(x-2)(2x+1)] = \ln x^2$

$(x-2)(2x+1) = x^2$

$2x^2 + x - 4x - 2 = x^2$

$x^2 - 3x - 2 = 0$

$x = \dfrac{3 \pm \sqrt{9 - 4(1)(-2)}}{2(1)}$

$= \dfrac{3 \pm \sqrt{17}}{2}$

only $\boxed{x = \dfrac{3+\sqrt{17}}{2}}$ is a solution

(c) $\frac{1}{2}\log(x+2) + \log 5 = 1$

$\log(x+2)^{1/2} + \log 5 = 1$

$\log[(x+2)^{1/2} \cdot 5] = 1$

$5\sqrt{x+2} = 10^1$

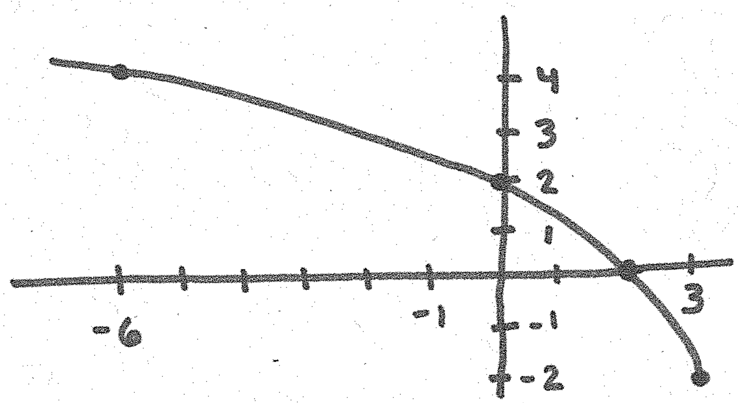
$\sqrt{x+2} = 2$

$x + 2 = 4$

$\boxed{x = 2}$

Question 7. *(4 marks)*
Sketch the function $y = -2 + 2\sqrt{3-x}$, and state its domain and range.

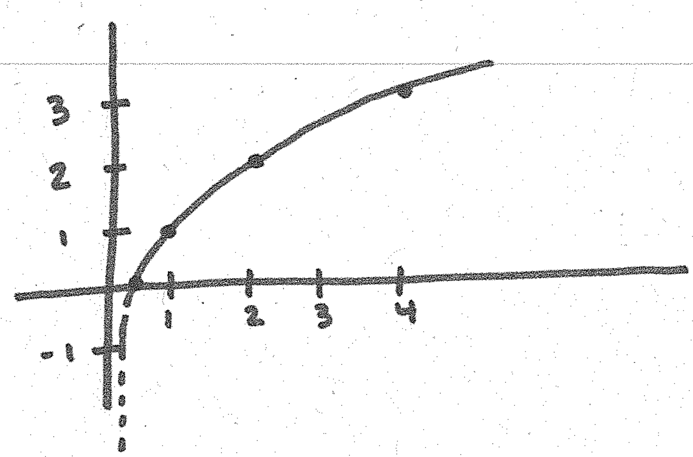
x	y
3	-2
2	0
-1	2
-6	4



Domain $x \leq 3$
Range $y \geq -2$

Question 8. *(4 marks)*
Sketch the function $y = 1 - \log_{\frac{1}{2}} x$, and state its domain and range.

x	y
1/4	-1
1/2	0
1	1
2	2
4	3



$\log_{\frac{1}{2}} x = 1-y$

$\left(\frac{1}{2}\right)^{1-y} = x$

Domain $x > 0$
Range $\mathbb{R}$

Question 9. *(4 marks)*
Given the quadratic function $y = 4x^2 - 12x - 7$, find
(a) The x-intercept(s)
(b) The y-intercept
(c) The vertex
(d) The domain and range
Note that you **do not have to sketch** a graph of the function

(a) y-intercept $(0,-7)$
(b) x-intercepts $0 = 4x^2 - 12x - 7$
$= 4x^2 - 14x + 2x - 7$
$= 2x(2x-7) + 1(2x-7)$
$= (2x+1)(2x-7)$
$x = -\frac{1}{2}, \frac{7}{2}$ $(-\frac{1}{2}, 0)$ & $(\frac{7}{2}, 0)$

(c) $x = -\frac{b}{2a} = \frac{12}{2(4)}$
$= \frac{12}{8} = \frac{3}{2}$
$y = 4(\frac{3}{2})^2 - 12(\frac{3}{2}) - 7 = 4(\frac{9}{4}) - 12(\frac{3}{2}) - 7$
$= 9 - 18 - 7 = -16$
$(\frac{3}{2}, -16)$

Domain $\mathbb{R}$
Range $[-16, \infty)$

BONUS *(2.5 marks)*
Solve the following equation

$x^{\log x} = 1000x^3$

$\log x^{\log x} = \log 1000 x^3$
$\log x \log x = \log 1000 + 3 \log x$
$(\log x)^2 - 3 \log x = 3$
$(\log x)^2 - 3\log x - 3 = 0$ let $u = \log x$
$u^2 - 3u - 3 = 0$
$u = \frac{3 \pm \sqrt{9 - 4(-3)}}{2} = \frac{3 \pm \sqrt{21}}{2}$ $\Longrightarrow$

$u = \frac{3+\sqrt{21}}{2}$

$\log x = \frac{3+\sqrt{21}}{2}$

so $\boxed{x = 10^{\frac{3+\sqrt{21}}{2}}}$

$u = \frac{3-\sqrt{21}}{2}$

$\log x = \frac{3-\sqrt{21}}{2}$

$\boxed{x = 10^{\frac{3-\sqrt{21}}{2}}}$

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