

NAME: Solutions

STUDENT NUMBER: _____

TEST 3

Dawson College
Applied Math (201-943-DW-S01)
Date: DECEMBER 5, 2014
Instructor: Emilie Richer

This test is scored out of 41 marks

Question 1. (2 marks each = 6 marks)

Perform the following operations involving complex numbers. Express your answers in simplest rectangular form.

a. $\sqrt{-9} + 2j^2 - 3\sqrt{-16}$

$$= 3j - 2 - 3(4j)$$

$$= 3j - 2 - 12j$$

$$= \boxed{-2 - 9j}$$

b. $\frac{1+j}{2-4j}$

$$= \frac{(1+j)(2+4j)}{(2-4j)(2+4j)} = \frac{2+4j+2j+4j^2}{4-16j^2} = \frac{-2+6j}{20} = \boxed{\frac{-1}{10} + \frac{3}{10}j}$$

c. $12j^{242} - (1+j^3)j^{15}$

$$= 12j^2 - j^{15} - j^{18}$$

$$= -12 - j^3 - j^2$$

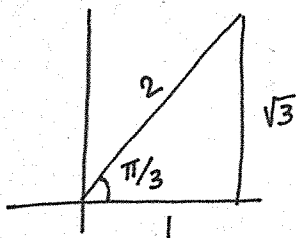
$$= -12 + j - 1$$

$$= \boxed{-11 + j}$$

Question 2. (6 marks)

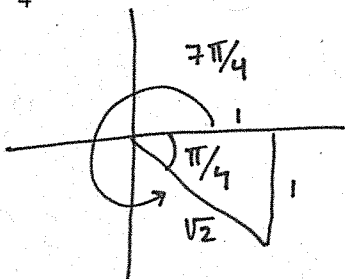
Use special triangles to find the **EXACT** values of the following.

a. $\tan \frac{\pi}{3}$



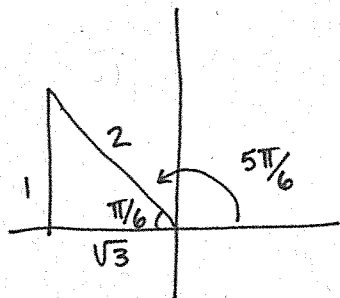
$$\tan \frac{\pi}{3} = \frac{\sqrt{3}}{1} \quad \left(\frac{y}{x} \right)$$

b. $\sin \frac{7\pi}{4}$



$$\sin \frac{7\pi}{4} = \frac{-1}{\sqrt{2}} \quad \left(\frac{y}{r} \right)$$

c. $\sec \frac{5\pi}{6}$



$$\sec \frac{5\pi}{6} = \frac{2}{-\sqrt{3}} \quad \left(\frac{r}{x} \right)$$

Question 4. (4 marks)

Solve the following equation for the real numbers a and b .

$$(a+bj)(7j-4) = 3j^5 + 16j^2$$

$$7aj - 4a + 7bj^2 - 4bj = 3j - 16$$

$$-4a - 7b + 7aj - 4bj = 3j - 16$$

EQUATIONS Real: $-4a - 7b = -16$ ①

$$7a - 4b = 3$$
 ②

isolate a in ① $a = 4 - \frac{7}{4}b$

sub in ②

$$7(4 - \frac{7}{4}b) - 4b = 3$$

$$28 - 49\frac{1}{4}b - 4b = 3$$

$$-65\frac{1}{4}b = -25$$

$$\frac{1}{4}b = \frac{20}{13}$$

$$a = 4 - \frac{7}{4}(\frac{20}{13})$$

$$a = \frac{17}{13}$$

Question 5. (5 marks)

Find all solutions to the trigonometric equation, in the range $0 \leq \theta < 2\pi$. Express your solutions in radian measurement.

$$3\sin\theta - 4\sin^2\theta = 0$$

$$\sin\theta(3 - 4\sin\theta) = 0$$

$$\sin\theta = 0 \quad \text{OR} \quad \sin\theta = \frac{3}{4}$$

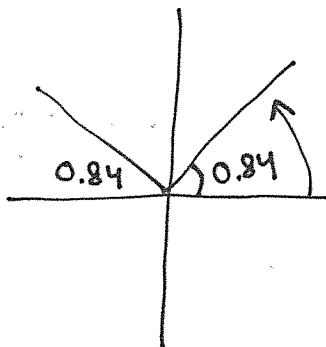
$$\sin\theta = 0$$

Two solutions:

$$\underline{0} \quad \& \quad \underline{\pi}$$

$$\sin\theta = \frac{3}{4}$$

$$\sin^{-1}(\frac{3}{4}) = 0.84$$



Two solution

$$\underline{0.84}$$

&

$$\underline{\pi - 0.84 = 2.29}$$

The solutions are $0, \pi, 0.84$ & 2.29

Question 6. (6 marks)

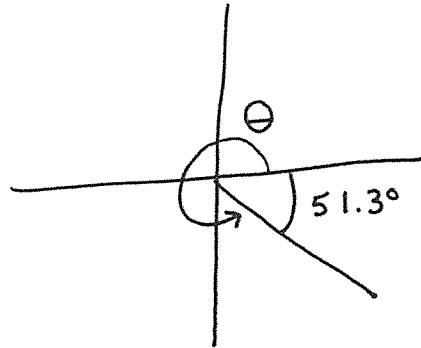
Find all solutions to the following trigonometric equation, subject to the listed constraints. Find all solutions in the range $0^\circ \leq \theta < 360^\circ$ expressing your solutions in degree measurement.

a. $2 \sec \theta = 3.2, \tan \theta < 0$

$$\sec \theta = \frac{3.2}{2}$$

$$\cos \theta = \frac{2}{3.2} = 0.625$$

$$\cos^{-1}(0.625) = 51.3^\circ$$

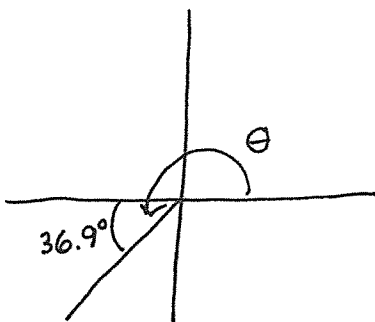


ONE QUADRANT WHERE
 $\cos \theta > 0$ +
& $\tan \theta < 0$ -

$$\begin{aligned} \theta &= 360^\circ - 51.3^\circ \\ &= \underline{308.7^\circ} \end{aligned}$$

b. $\tan \theta = 0.75, \sec \theta < 0$

$$\tan^{-1}(0.75) = 36.9^\circ$$

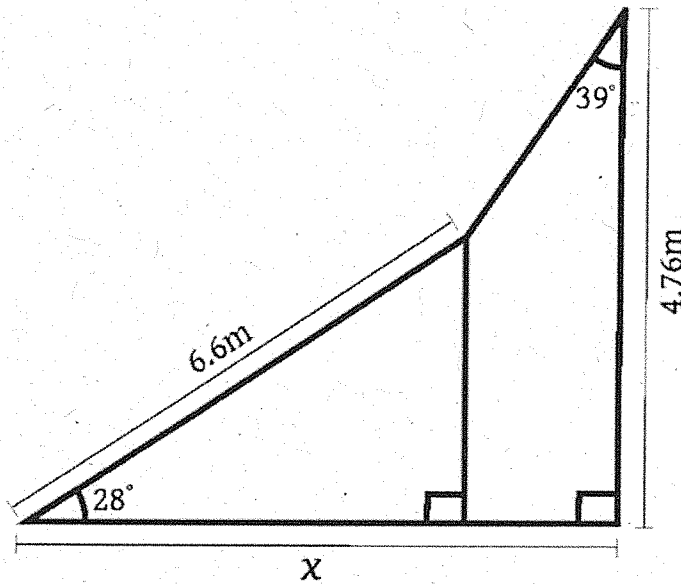


ONE QUADRANT
WHERE $\cos \theta$ -
& $\tan \theta$ +

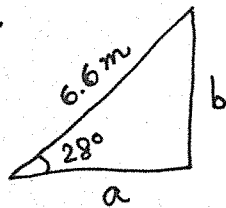
$$\begin{aligned} \theta &= 180^\circ + 36.9^\circ \\ &= \underline{216.9^\circ} \end{aligned}$$

Question 7. (4 marks)

Find the value of x in the diagram below.



triangle

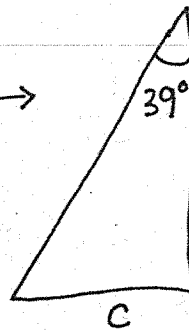
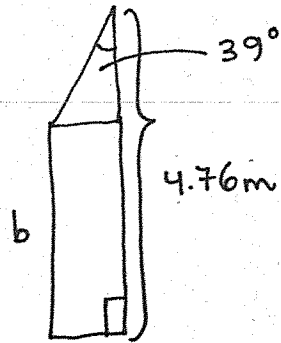


$$\sin 28^\circ = \frac{b}{6.6}$$

$$b = 3.1 \text{ m}$$

$$\cos 28^\circ = \frac{a}{6.6}$$

$$a = 5.83 \text{ m}$$



$$4.76 - 3.1 = 1.66 \text{ m}$$

$$\tan 39^\circ = \frac{c}{1.66}$$

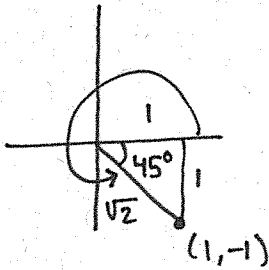
$$c = 1.66 \tan 39^\circ = 1.34$$

$$x = a + c = 5.83 + 1.34$$

$$= \boxed{7.17 \text{ m}}$$

Question 8. (6 marks)

a. Express the complex number $1 - j$ in polar and exponential forms.



$$\theta = 315^\circ, \frac{7\pi}{4}$$

$$r = \sqrt{2}$$

$$\text{polar: } \sqrt{2} / \underline{315^\circ}$$

$$\text{exp: } \sqrt{2} e^{\frac{7\pi}{4}j}$$

b. Express the complex number $(1 - j)^{16}$ in rectangular form.

$$(1-j)^{16} = \left(\sqrt{2} e^{\frac{7\pi}{4}j}\right)^{16}$$

$$= (\sqrt{2})^{16} e^{28\pi j}$$

$$= 256 e^{28\pi j}$$

$$28\pi = 5040^\circ$$

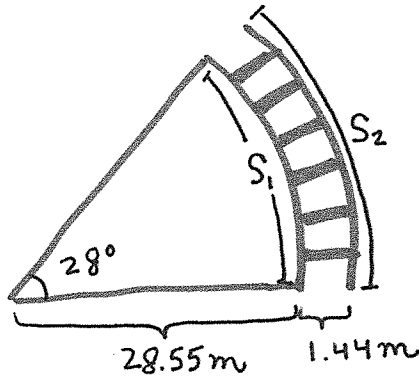
$$\text{polar } 256 \cos 5040^\circ + 256 \sin 5040^\circ j = 256 / \underline{5040^\circ}$$

$$\text{rectangular } 256(1) + 256(0)j$$

$$= 256$$

Question 9. (4 marks)

Part of a railway track follows a circular arc with a central angle of 28° . If the radius of the arc of the inner rail is 28.55m and the rails are 1.44m apart, how much longer is the outer rail than the inner rail?



$$28^\circ = \frac{7\pi}{45} \text{ (rads)}$$

$$\theta = \frac{S_1}{r_i} \quad \left. \vphantom{\theta} \right\} \text{ inner rail}$$

$$\frac{7\pi}{45} = \frac{S_1}{28.55} \quad S_1 = 13.95 \text{ m}$$

$$\theta = \frac{S_2}{r_{\text{outer}}} \quad \left. \vphantom{\theta} \right\} \text{ outer rail}$$

$$\frac{7\pi}{45} = \frac{S_2}{29.99} \quad r_{\text{outer}} = 28.55 + 1.44 = 29.99 \text{ m}$$

$$S_2 = 14.66$$

$$\begin{aligned} \text{Difference } S_2 - S_1 &= 14.66 - 13.95 \\ &= \underline{0.71 \text{ m}} \end{aligned}$$

BONUS (2 marks)

Solve the following equation

$$x^{\log x} = 1000x^3$$

see test #2
