

Quiz 11

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (2 marks) §8.1 #7 Find a formula for the general term a_n of the sequence, assuming that the pattern of the first few terms continues.

$$\left\{ \frac{1}{2}, -\frac{4}{3}, \frac{9}{4}, -\frac{16}{5}, \frac{25}{6}, \dots \right\} \quad a_n = (-1)^{n+1} \frac{n^2}{n+1}$$

Question 2. (3 marks) §8.1 #31 Determine whether the sequence converges or diverges. If it converges, find the limit.

$$\begin{aligned} a_n &= \ln(2n^2 + 1) - \ln(n^2 + 1) \\ &= \ln \left(\frac{2n^2 + 1}{n^2 + 1} \right) \end{aligned} \quad \begin{aligned} \lim_{n \rightarrow \infty} a_n &= \lim_{n \rightarrow \infty} \ln \left(\frac{2n^2 + 1}{n^2 + 1} \right) \\ &= \lim_{x \rightarrow \infty} \ln \left(\frac{2x^2 + 1}{x^2 + 1} \right) \\ &= \ln \left(\lim_{x \rightarrow \infty} \frac{2x^2 + 1}{x^2 + 1} \right) = \ln 2 \end{aligned}$$

Question 3. (5 marks) §8.2 #20 Determine whether the series is convergent or divergent. If it is convergent, find its sum.

$$\begin{aligned} &\sum_{n=1}^{\infty} [(0.8)^{n-1} - (0.3)^n] \\ &\sum_{n=1}^{\infty} (0.8)^{n-1} \quad |r| = 0.8 < 1 \quad \therefore \text{converges} \\ &= \frac{a}{1-r} = \frac{1}{1-0.8} = \frac{1}{0.2} = 5 \\ &\sum_{n=1}^{\infty} (0.3)^n = \sum_{n=0}^{\infty} (0.3)^n - a_0 \quad |r| = 0.3 < 1 \\ &= \frac{1}{1-0.3} - 1 \\ &= \frac{1}{0.7} - 1 \\ &= \frac{10}{7} - \frac{7}{7} \\ &= \frac{3}{7} \end{aligned}$$

$$\begin{aligned} S_0 &= \sum_{n=1}^{\infty} [(0.8)^{n-1} - (0.3)^n] \\ &= \sum_{n=1}^{\infty} (0.8)^{n-1} - \sum_{n=1}^{\infty} (0.3)^n \\ &= 5 - \frac{3}{7} = \frac{35-3}{7} = \frac{32}{7} \end{aligned}$$