Dawson College: Calculus II (SCIENCE): 201-NYB-05-S03: Fall 2014

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Quiz 4

This quiz is graded out of 15 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §5.2 #21 Use the form of the definition of the integral given to evaluate the integral.

$f(x) = x^2 - 4x + 2$

$\Delta x = \frac{b-a}{n} = \frac{4-1}{n} = \frac{3}{n}$

$x_i = a + i\Delta x$
$= 1 + \frac{3i}{n}$

$$\int_1^4 (x^2 - 4x + 2)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f\left(1+\frac{3i}{n}\right)\frac{3}{n}$$

$$= \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} \left[\left(1+\frac{3i}{n}\right)^2 - 4\left(1+\frac{3i}{n}\right) + 2\right]$$

$$= \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} \left[1 + \frac{6i}{n} + \frac{9i^2}{n^2} - 4 - \frac{12i}{n} + 2\right]$$

$$= \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} \left[\frac{9i^2}{n^2} - \frac{6i}{n} - 1\right]$$

$$= \lim_{n\to\infty} \frac{3}{n} \left[\frac{9}{n^2}\sum_{i=1}^{n} i^2 - \frac{6}{n}\sum_{i=1}^{n} i - \sum_{i=1}^{n} 1\right]$$

$$= \lim_{n\to\infty} \left[\frac{\cancel{27}^{\,9}}{n^{\cancel{3}\,2}} \frac{\cancel{n}(n+1)(2n+1)}{\cancel{6}\,2} - \frac{\cancel{18}^{\,9}}{n^{\cancel{2}}} \frac{\cancel{n}(n+1)}{\cancel{2}} - \frac{3\cancel{n}}{\cancel{n}}\right]$$

$$= \lim_{n\to\infty} \left[\frac{9(n+1)}{2n} \cdot \frac{(2n+1)}{n} - \frac{9(n+1)}{n} - 3\right]$$

$$= \frac{9}{\cancel{2}} \cdot \frac{\cancel{2}}{1} - \frac{9}{1} - 3$$

$$= -3$$

Question 2. *(5 marks)*§5.3 #29 Evaluate the integral.

$$\int_{-1}^{2} (x - 2|x|)\, dx$$

$$= \int_{-1}^{2} x\, dx - 2\int_{-1}^{2} |x|\, dx$$

$$= \left[\frac{x^2}{2}\right]_{-1}^{2} - 2\left[\int_{-1}^{0} |x|\, dx + \int_{0}^{2} |x|\, dx\right]$$

$$= \frac{2^2}{2} - \frac{(-1)^2}{2} - 2\left[\int_{-1}^{0} -x\, dx + \int_{0}^{2} x\, dx\right]$$

$$= 2 - \frac{1}{2} - 2\left[\left[-\frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_{0}^{2}\right]$$

$$= 2 - \frac{1}{2} - 2\left[\frac{(-1)^2}{2} + \frac{2^2}{2}\right]$$

$$= \frac{3}{2} - 2\left[\frac{5}{2}\right] = -\frac{7}{2}$$

Question 3. *(5 marks)*§5.4 #19

a. *(2 marks)* Find the average value of f on the given interval.

b. *(2 marks)* Find c such that $f_{ave} = f(c)$.

c. *(1 mark)* Sketch the graph of f and a rectangle whose area is the same as the area under the graph of f.

$f(x) = (x-3)^2, \quad [2,5]$

a) $$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\, dx = \frac{1}{5-2}\int_2^5 (x-3)^2\, dx = \frac{1}{3}\int_2^5 x^2 - 6x + 9\, dx = \frac{1}{3}\left[\frac{x^3}{3} - \frac{6x^2}{2} + 9x\right]_2^5$$

$$= \frac{1}{3}\left[\left[\frac{5^3}{3} - \frac{6\,5^2}{2} + 9(5)\right] - \left[\frac{2^3}{3} - \frac{6(2)^2}{2} + 9(2)\right]\right] = \frac{1}{3}\left[\frac{125}{3} - 75 + 45 - \frac{8}{3} + 12 - 18\right]$$

$$= \frac{1}{3}\left[\frac{117}{3} - 36\right] = 1$$

b) $1 = f(c)$

$1 = (c-3)^2$

$\pm\sqrt{1} = (c-3)$

$c = 3 \pm 1 = 2$ and 4

c)