

Quiz 5

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (4 marks) §5.5 #35 Evaluate the indefinite integral.

$$\int \frac{1+x}{1+x^2} dx = \int \frac{1}{1+x^2} dx + \int \frac{x}{1+x^2} dx$$

$$= \arctan x + \int \frac{1}{u} \frac{du}{2}$$

$$= \arctan x + \frac{1}{2} \ln|u| + C$$

$$= \arctan x + \frac{1}{2} \ln(1+x^2) + C$$

$$u = 1+x^2 \\ du = 2x dx \\ \frac{du}{2} = x dx$$

Question 2. (3 marks) §5.5 #46 Evaluate the definite integral.

$$\int_{-\pi/2}^{\pi/2} \frac{x^2 \sin x}{1+x^6} dx = 0$$

$$\text{Let } f(x) = \frac{x^2 \sin x}{1+x^6} \text{ then}$$

$$f(-x) = \frac{(-x)^2 \sin(-x)}{1+(-x)^6} = \frac{x^2 (-\sin x)}{1+x^6} = -f(x).$$

$\therefore f(x)$ is an odd func.
 \therefore by thm. seen in class

Question 3. (3 marks) §5.5 #52 Evaluate the definite integral.

$$\int_0^{1/2} \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx = \int_0^{\pi/6} u du = \left[\frac{u^2}{2} \right]_0^{\pi/6} = \frac{\pi^2}{72}.$$

$$u = \arcsin x$$

$$du = \frac{1}{\sqrt{1-x^2}} dx$$

$$u(1/2) = \arcsin(1/2) = \pi/6$$

$$u(0) = \arcsin 0 = 0$$