

## Quiz 7

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

**Question 1.** (5 marks) §6.2 #45 Evaluate the definite integral.

$$\int_0^a \frac{dx}{(a^2+x^2)^{3/2}}$$

**Question 2.** (5 marks) §6.3 #19 Evaluate the indefinite integral.

$$\int \frac{x^2+1}{(x-3)(x-2)^2} dx$$

$$x = a \tan \theta$$

$$dx = a \sec^2 \theta d\theta$$

$$0 = a \tan \theta$$

$$0 = \theta$$

$$a = a \tan \theta$$

$$\frac{\pi}{4} = \theta$$

$$\int_0^{\pi/4} \frac{a \sec^2 \theta d\theta}{(a^2 + a^2 \tan^2 \theta)^{3/2}}$$

$$= \int_0^{\pi/4} \frac{a \sec^2 \theta}{(a^2 (1 + \tan^2 \theta))^{3/2}} d\theta$$

$$= \int_0^{\pi/4} \frac{a \sec^2 \theta}{(a^2 \sec^2 \theta)^{3/2}} d\theta$$

$$= \int_0^{\pi/4} \frac{a \sec^2 \theta}{a^3 \sec^3 \theta} d\theta$$

$$= \frac{1}{a^2} \int_0^{\pi/4} \frac{1}{\sec \theta} d\theta$$

$$= \frac{1}{a^2} \int_0^{\pi/4} \cos \theta d\theta$$

$$= \frac{1}{a^2} [\sin \theta]_0^{\pi/4}$$

$$= \frac{1}{a^2} [\sin \pi/4 - \sin 0]$$

$$= \frac{1}{a^2} [\frac{1}{\sqrt{2}} - 0]$$

$$= \frac{1}{\sqrt{2} a^2}$$

$$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{A}{x-3} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

$$x^2+1 = A(x-2)^2 + B(x-3)(x-2) + C(x-3)$$

Let  $x = 2$

$$2^2+1 = A(2-2)^2 + B(2-3)(2-2) + C(2-3)$$

$$C = -5$$

Let  $x = 3$

$$3^2+1 = A(3-2)^2 + B(3-3)(3-2) + C(3-3)$$

$$A = 10$$

Let  $x = 0$

$$0^2+1 = A(0-2)^2 + B(0-3)(0-2) + C(0-3)$$

$$1 = 4(10) + 6B + (-5)(-3)$$

$$-54 = 6B$$

$$B = -9$$

$$\int \left( \frac{10}{x-3} - \frac{9}{x-2} - \frac{5}{(x-2)^2} \right) dx$$

$$= 10 \ln|x-3| - 9 \ln|x-2| + \frac{5}{x-2} + C$$