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Quiz 8

This quiz is graded out of 15 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §3.7 #25 Evaluate the ~~definite integral~~ limit

$$\lim_{x\to\infty} x^3 e^{-x^2} = \lim_{x\to\infty} \frac{x^3}{e^{x^2}} \overset{\hat{H}}{=} \lim_{x\to\infty} \frac{3x^2}{e^{x^2}2x} \overset{\hat{H}}{=} \lim_{x\to\infty} \frac{6x}{2e^{x^2}+4x^2e^{x^2}}$$

$$\overset{\hat{H}}{=} \lim_{x\to\infty} \frac{6}{4xe^{x^2}+8xe^{x^2}+8x^3e^{x^2}} = 0$$

Question 2. *(5 marks)* §6.6 #19 Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

$$\int_{-\infty}^{\infty} \cos(\pi t)\,dt = \int_{-\infty}^{0} \cos(\pi t)dt + \int_{0}^{\infty} \cos(\pi t)dt$$

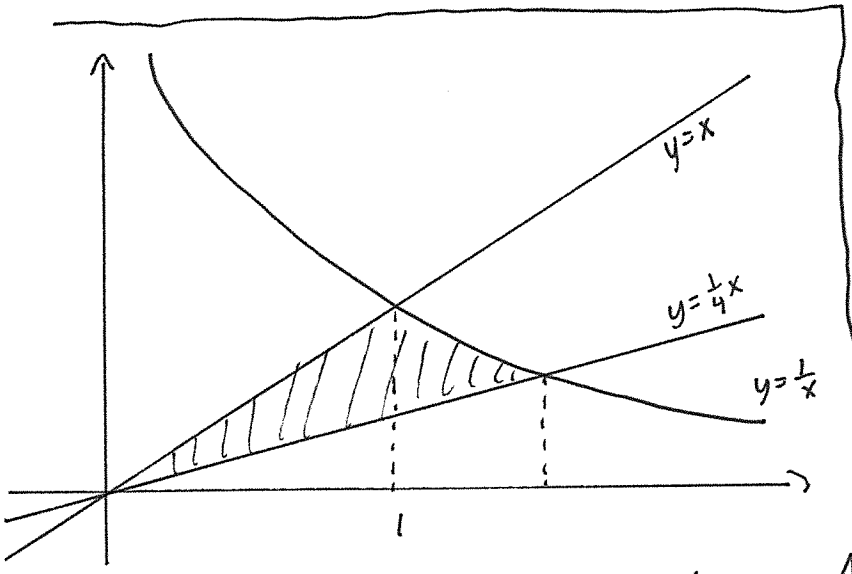
$$= \lim_{a\to-\infty} \int_a^0 \cos(\pi t)dt + \lim_{b\to\infty} \int_0^b \cos(\pi t)dt = \lim_{a\to-\infty} \left[\frac{\sin(\pi t)}{\pi}\right]_a^0 + \lim_{b\to\infty} \left[\frac{\sin(\pi t)}{\pi}\right]_0^b$$

$$= \lim_{a\to-\infty} -\frac{\sin(\pi a)}{\pi} + \lim_{b\to\infty} \frac{\sin(\pi b)}{\pi}$$

both limit oscilate between -1 and 1. Does not converge

Question 3. *(5 marks)* §7.1 #19 Sketch the region enclosed by the given curves and find its area.

$y = 1/x,\ y = x,\ y = \frac{1}{4}x,\ x > 0$



Intersection between $y = x$ and $y = \frac{1}{x}$

$x = \frac{1}{x}$
$x^2 = 1$
$x = \pm 1$ so $x = 1$

Intersection between $y = \frac{1}{4}x$ and $y = x$

$\frac{1}{4}x = x$
$\frac{3}{4}x = 0$
$x = 0$

Intersection between $y = \frac{1}{4}x$ and $y = \frac{1}{x}$

$\frac{1}{4}x = \frac{1}{x} \Leftrightarrow x^2 = 4 \Leftrightarrow x = \pm 2$ so $x = 2$

$$A = \int_0^1 \overbrace{x - \tfrac{1}{4}x}^{\frac{3}{4}x}\,dx + \int_1^2 \frac{1}{x} - \frac{1}{4}x\,dx$$

$$= \left[\frac{3}{8}x^2\right]_0^1 + \left[\ln|x| - \frac{1}{8}x^2\right]_1^2$$

$$= \frac{3}{8} + \ln 2 - \frac{1}{8}2^2 - \ln 1 + \frac{1}{8}$$

$$= \ln 2$$