Dawson College: **Linear Algebra**: 201-NYC-05-S05: Fall 2014

Name: ____________________
Student ID: ____________________

Test 1

This test is graded out of 55 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1.

a. *(6 marks)* Solve the following system by Gauss-Jordan elimination:

$$\begin{array}{rcrcrcrcr} 2x_1 & - & x_2 & + & x_3 & - & 3x_4 & = & 3 \\ 3x_1 & + & 3x_2 & - & x_3 & + & x_4 & = & 5 \\ 5x_1 & + & 2x_2 & & & - & 2x_4 & = & 8 \\ & & & & x_3 & - & x_4 & = & 1 \end{array}$$

b. *(2 marks)* Find two particular solution to the above system.

Question 2. Consider the matrices:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 2 & 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 2 & 2 & 1 \\ 0 & -4 & 0 \end{bmatrix} C = \begin{bmatrix} 0 & 1 \\ -3 & 0 \\ 3 & -1 \end{bmatrix} D = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix}$$

a. *(1 mark)* Compute the following, if possible.

$$C - D$$

b. *(2 marks)* Compute the following, if possible.

$$(BC)^t$$

c. *(2 marks)* Compute the following, if possible.

$$A^2$$

d. *(2 marks)* Compute the following, if possible.

$$\mathrm{tr}(A^2 + CB)$$

g. *(5 marks)* Find E, if possible.

$$(2I - (DE)^t)^{-1} = (BC)^t$$

Question 3. *(4 marks)* Given the following augmented matrix in row-echelon form, solve the system using back substitution.

$$\begin{bmatrix} 1 & 5 & -3 & 2 & 4 & 0 \\ 0 & 0 & 1 & -2 & 3 & -1 \\ 0 & 0 & 0 & 1 & 2 & 2 \end{bmatrix}$$

Question 4.[1] Let A and B be $n \times n$ invertible matrices, and AB is its own inverse (i.e. $(AB)^{-1} = AB$).

a. *(2 marks)* Prove that BA is invertible and is its own inverse.

b. *(2 marks)* Evaluate and simplify $(AB+I)^2$

c. *(2 marks)* Evaluate and simplify $(AB+I)^8$

[1]From a John Abbott Final Examination

Question 5. Let A and B be $n \times n$ matrices. Prove that

a. *(2 marks)* if $AB = BA$ then $A^t B^t = B^t A^t$

b. *(2 marks)* if $A^t B^t = B^t A^t$ then $AB = BA$

Question 6.[2] Let

$$A = \begin{bmatrix} 2 & 1 \\ x & y \end{bmatrix}$$

a. *(2 marks)* Find x and y such that $A^2 = 0$, if possible.

b. *(2 marks)* Find x and y such that $A^2 = I$, if possible.

[2]From a John Abbott Final Examination

Question 7. Consider the following system:

$$\begin{array}{ccccccc} x & + & y & + & 7z & = & -1 \\ x & + & 2y & + & 3z & = & 3 \\ 2x & + & 3y & + & (a^2+11)z & = & 3a+5 \end{array}$$

where $a \in \mathbb{R}$, determine the values of a so that the system has

a. *(2 marks)* a unique solution, justify.

b. *(2 marks)* infinitely many solutions, justify.

c. *(2 marks)* no solutions, justify.

Bonus Question. *(5 marks)* Let $M = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$, where A, B, C, and D are all $n \times n$ matrices and each commutes with all the others. If $M^2 = \mathbf{0}$, prove that $(A+D)^3 = \mathbf{0}$.