

Name: \_\_\_\_\_  
Student ID: \_\_\_\_\_

## Test 2

This test is graded out of 40 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

**Question 1.** (5 marks) Evaluate the integral.

$$\int_0^{\pi/6} \sqrt{1 + \cos 2x} \, dx$$

**Question 2.** (5 marks) Evaluate the integral.

$$\int \sqrt{5+4x-x^2} dx$$

**Question 3.** (5 marks) Evaluate the integral.

$$\int \frac{x^2 + 1}{(x-3)(x-2)^2} dx$$

**Question 4.** (5 marks) Set up the integral to find the volume of the solid obtained from the region bounded by the graphs of  $y = \operatorname{arcsec}(x + 1)$ ,  $y = \frac{\pi}{4}$ ,  $x = 0$  rotated about the line  $x = e$ . Sketch the region, draw a representative rectangle, write a representative element and the integral.

**Question 5.** (5 marks) Determine whether the integral is convergent or divergent. Evaluate the integral if convergent.

$$\int_0^1 \frac{dx}{\sqrt{1-x^2}}$$

**Question 6.** (5 marks) Evaluate the integral.

$$\int_1^{\sqrt{3}} \arctan\left(\frac{1}{x}\right) dx$$

**Question 7.** (5 marks) Use the Comparison Theorem to determine whether the integral is convergent or divergent.

$$\int_0^{\infty} \frac{\arctan x}{2 + e^x} dx$$

**Question 8.** (5 marks) Find the value(s) of  $c$  such that the area of the region bounded by the parabolas  $y = x^2 - c^2$  and  $y = c^2 - x^2$  is 1936.



**Bonus Question.** (3 marks) Given that  $\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \int_e^x \frac{f(t)}{t} dt = 0$  and  $\lim_{x \rightarrow \infty} \int_e^x f(t) dt = \infty$  determine whether the integral is convergent or divergent. Evaluate the integral if convergent.

$$\int_e^\infty \frac{1}{z^2} \int_e^z f(x) dx dz$$