**Name:** _____

**Student ID:** _____

# Test 3

This test is graded out of 40 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

**Question 1.** *(5 marks)* Determine whether the series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=0}^{\infty} \left[ \pi \left( \frac{e}{\pi} \right)^n + \left( \frac{e}{n+1} - \frac{e}{n+2} \right) \right]$$

**Question 2.** *(5 marks)* Determine whether the series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=2014}^{\infty} \sqrt[n]{2}$$

**Question 3.** *(5 marks)* Determine whether the series is convergent or divergent.

$$\sum_{n=0}^{\infty} \frac{1+\cos n}{2^n}$$

**Question 4.** *(5 marks)* Set up the integral to find the volume of the solid obtained from the region bounded by the graphs of $y = xe^{-x}$, $y = 0$, $x = 2$ rotated about the line $x = e$. Sketch the region, draw a representative rectangle, write a representative element and the integral.

**Question 5.** *(5 marks)* Find the exact length of the curve

$$y = \frac{x^3}{3} + \frac{1}{4x}, \quad 1 \le x \le 2$$

**Question 6.**

a. *(2 marks)* Find a formula for the general term $a_n$ of the sequence, assuming that the pattern of the first few terms continues.

$$\left\{ \frac{2}{1}, \frac{4}{1(2)}, \frac{8}{1(2)(3)}, \frac{16}{1(2)(3)(4)}, \frac{32}{1(2)(3)(4)(5)}, \cdots \right\}$$

b. *(3 marks)* Determine whether the sequence in part a. converges or diverges. If it converges, find the limit.

**Question 7.** *(5 marks)* Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=0}^{\infty} \frac{(-3)^n}{(2n+1)!}$$

**Question 8.** *(5 marks)* Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$$

**Bonus Question.** *(3 marks)* Prove: If $\int_0^1 f(x)\, dx = \lim_{b \to \infty} \int_0^b f(x)\, dx$ then

$$\sum_{n=1}^{\infty} \int_n^{n+1} f(x)\, dx = 0$$