Dawson College: **Calculus II** (SCIENCE): 201-NYB-05-S03: Fall 2014

Name: ____________________
Student ID: ____________________

Test 1

This test is graded out of 40 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

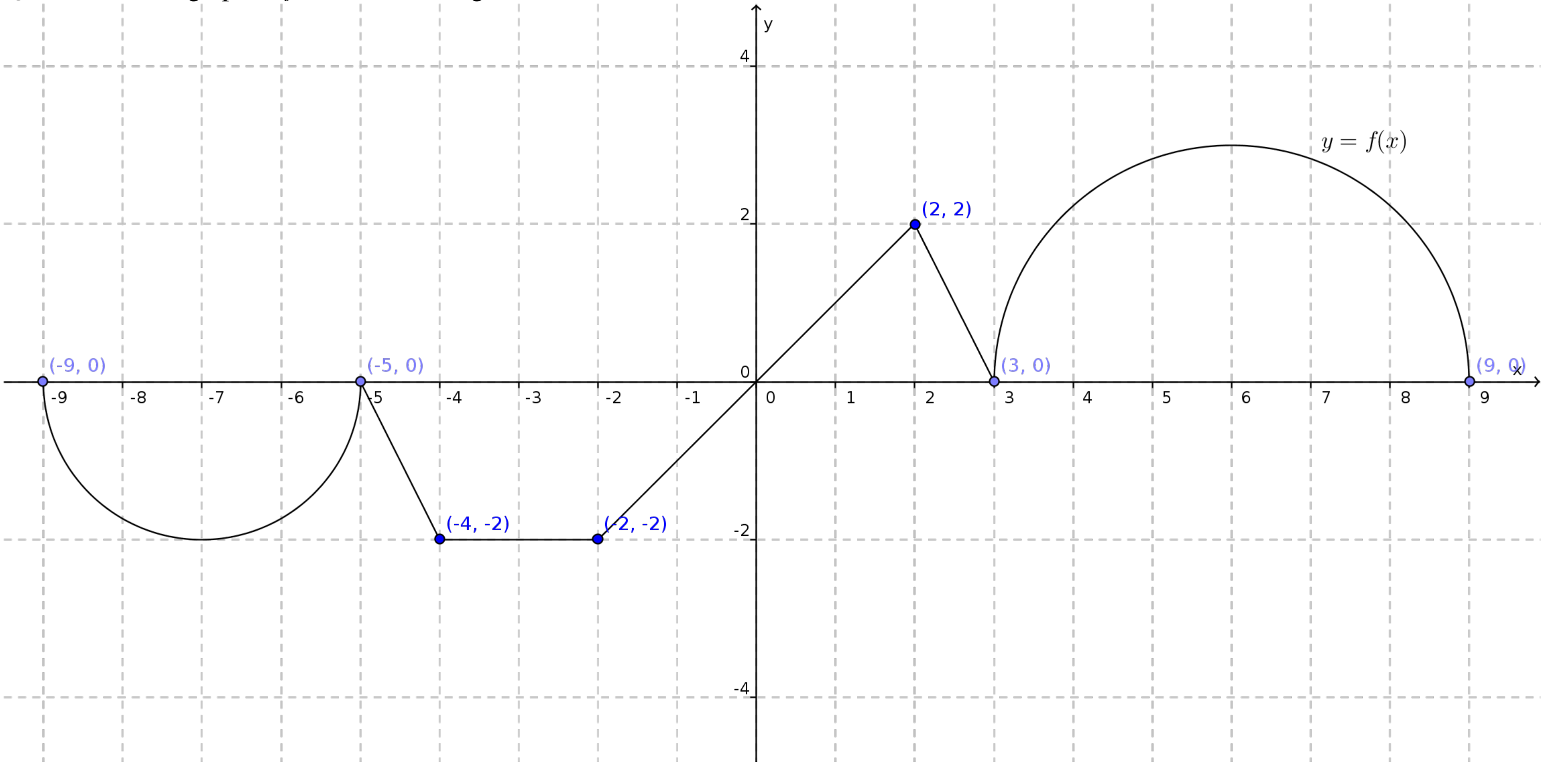
Formulae:

$\sum_{i=1}^{n} c = cn$ where c is a constant $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \quad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

Question 1. *(5 marks)* Evaluate the definite integral of $f(x) = -x^2 + x + 1$ on $[-1, 2]$ using the definition of the definite integral.

Question 2. The graph of f consists of straight lines and two semicircles.



a. *(3 marks)* Find the Riemann sum for $f(x)$, if the partition points are -9, -5, -2, 2, 3, 9 and the sample points are -7, -4, 1, 2.5, 6.

b. *(1 mark)* Draw the approximating rectangles.

Question 3. *(5 marks)* If

$$\int_2^3 (f(x) - 1)\, dx = 2, \quad \int_2^1 3f(x)\, dx = 4,$$

and $f(-x) = -f(x)$ for all $x \in \mathbb{R}$ find

$$\int_{-1}^3 f(x)\, dx.$$

Question 4. *(5 marks)* Evaluate the definite integral:

$$\int_0^{1/2} \frac{\arcsin x}{\sqrt{1-x^2}}\,dx$$

Question 5. Given

$$h(x) = \int_{\arcsin \pi x}^{\sin e^x} (\arcsin u)\sin u\,du$$

a. *(2 marks)* Rewrite $h(x)$ as the sum of two integrals with a constant as the lower bound.

b. *(1 mark)* Rewrite the two integrals of part a. as composite functions with an integral as the outer function.

c. *(2 marks)* Using part b. and the 2^{nd} FTC determine $h'(x)$.

Question 6. Given

$$f(x) = \sec x - 1, \quad [0, \pi/3]$$

a. *(2 marks)* Find the average value of f on the given interval.

b. *(2 marks)* Find c such that $f_{ave} = f(c)$.

c. *(2 marks)* Sketch the graph of f and a rectangle whose area is the same as the area under the graph of f.

Question 7. *(5 marks)* Prove: If f is an odd continuous function on $[-a,a]$, then

$$\int_{-a}^{a} f(x)\,dx = 0$$

Question 8. *(5 marks)* Evaluate the indefinite integral

$$\int \frac{e^x + e^{2x}}{1+e^{2x}}\,dx$$

Bonus Question. *(3 marks)*
The function F is defined on $[-9,9]$ as

$$F(z) = \int_{-9}^{z} f(x)\, dx.$$

where $f(x)$ is the graph of **Question 2**. Discuss where the second derivative test is conclusive and where it fails. *(Hint: use the graph of $f(x)$)*