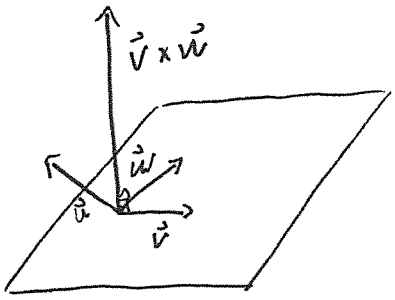
Name: Y. Lamontagne
Student ID: _____

Quiz 10

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §3.5 #35 Show that if $\vec{u},\vec{v}$, and $\vec{w}$ are vectors in $\mathbf{R}^3$, no two of which are collinear, then $\vec{u}\times(\vec{v}\times\vec{w})$ lies in the plane determined by $\vec{v}$ and $\vec{w}$.



Then $\vec{u}\times(\vec{v}\times\vec{w})$ lies on the plane

if $\vec{u}\times(\vec{v}\times\vec{w})\cdot(\vec{v}\times\vec{w}) = 0$

Let $\vec{y} = (y_1, y_2, y_3) = \vec{v}\times\vec{w}$

then $(\vec{u}\times\vec{y})\cdot\vec{y} = \vec{y}\cdot(\vec{u}\times\vec{y})$

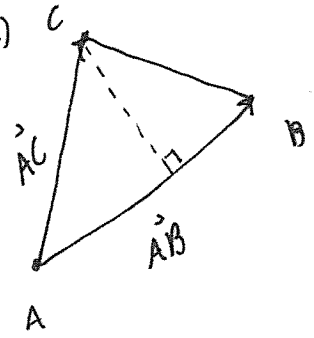
$$= \begin{vmatrix} y_1 & y_2 & y_3 \\ u_1 & u_2 & u_3 \\ y_1 & y_2 & y_3 \end{vmatrix} = 0$$

Question 2. §3.5 #27

a. *(3 marks)* Find the area of the triangle having vertices $A(1,0,1)$, $B(0,2,3)$, and $C(2,1,0)$.

b. *(2 marks)* Use the result of part a. to find the length of the altitude from vertex C to side AB.

a)



$\vec{AC} = C - A = (2,1,0) - (1,0,1) = (1,1,-1)$

$\vec{AB} = B - A = (0,2,3) - (1,0,1) = (-1,2,2)$

$$\vec{AB}\times\vec{AC} = \begin{matrix} -1 & 1 \\ 2 & 1 \\ 2 & -1 \end{matrix} = \left(\begin{vmatrix} 2 & 2 \\ 1 & -1 \end{vmatrix}, -\begin{vmatrix} -1 & 2 \\ 1 & -1 \end{vmatrix}, \begin{vmatrix} -1 & 2 \\ 1 & 1 \end{vmatrix}\right) = (4,-1,3)$$

$$\text{Area} = \frac{\|\vec{AB}\times\vec{AC}\|}{2} = \frac{\sqrt{4^2+(-1)^2+3^2}}{2}$$

b) $\text{Area} = \frac{\text{base}\,(\text{altitude})}{2}$ $= \frac{\sqrt{26}}{2}$

$$\frac{\sqrt{26}}{2} = \frac{\|\vec{AB}\|\ \text{altitude}}{2}$$

$$\text{altitude} = \frac{\sqrt{26}}{3}$$