Dawson College: Linear Algebra (SCIENCE): 201-NYC-05-S4: Fall 2014

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Quiz 12

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §4.3 #15 Show that if $\{\vec{v}_1, \vec{v}_2\}$ is linearly independent and $\vec{v}_3$ does not lie in span$\{\vec{v}_1, \vec{v}_2\}$, then $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly independent.

Suppose $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly dependent then there exists non zero K_i s.t. $K_1\vec{v}_1 + K_2\vec{v}_2 + K_3\vec{v}_3 = \vec{0}$. Suppose one $K_i \neq 0$, this implies that $\vec{v}_1, \vec{v}_2$ or $\vec{v}_3$ is $\vec{0}$. But $\vec{v}_3 \notin \text{span}\{\vec{v}_1, \vec{v}_2\}$ $\therefore$ $V_3 \neq 0$. But $\{\vec{v}_1, \vec{v}_2\}$ is linearly independent $\therefore$ $\vec{V}_1 \neq 0 \neq \vec{V}_2$. ↯

Suppose two $K_i \neq 0$. $K_1 \neq 0 \neq K_2$ ↯ since $\{\vec{v}_1, \vec{v}_2\}$ is linearly independent.
$K_2 \neq 0 \neq K_3$ ↯ since $\vec{V}_3 \notin \text{span}\{\vec{V}_1, \vec{V}_2\}$
$K_1 \neq 0 \neq K_3$ ↯ since $V_3 \notin \text{span}\{\vec{V}_1, \vec{V}_2\}$

Suppose three $K_i \neq 0$ ↯ since $V_3 \notin \text{span}\{\vec{V}_1, \vec{V}_2\}$

$\therefore K_i = 0 \ \forall i$ $\therefore$ $\{\vec{V}_1, \vec{V}_2, \vec{V}_3\}$ is linearly independent.

Question 2. *(5 marks)* §4.4 #4d Is the following a basis for P_2?

$\{-4+x+3x^2, 6+5x+2x^2, 8+4x+x^2\}$

① Is the set linearly independent? $K_1(-4+x+3x^2) + K_2(6+5x+2x^2) + K_3(8+4x+x^2) = \vec{0}$

$$\begin{bmatrix} -4 & 6 & 8 \\ 1 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

K_i has only the trivial solution if $|A| \neq 0$

$$|A| = -4(5)(1) + 6(4)(3) + 8(1)(2) - 8(5)(3) - (-4)(4)(2) - 1(1)(6) = -20 + 72 + 16 - 120 + 32 - 6 = -26 \neq 0$$

② Does the set span P_2? Let $a + bx + cx^2 \in P_2$

$K_1(-4+x+3x^2) + K_2(6+5x+2x^2) + K_3(8+4x+x^2) = a + bx + cx^2$

$$\begin{bmatrix} -4 & 6 & 8 \\ 1 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

since $|A| \neq 0$. a unique solution exists for any a, b, c.

$\therefore$ the set spans P_2

$\therefore$ the set is a basis for P_2.