Dawson College: Linear Algebra (SCIENCE): 201-NYC-05-S4: Fall 2014

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Quiz 12

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §4.3 #15 Show that if $\{\vec{v}_1, \vec{v}_2\}$ is linearly independent and $\vec{v}_3$ does not lie in span$\{\vec{v}_1, \vec{v}_2\}$, then $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly independent.

Suppose $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly dependent then there exists non zero k_i s.t. $k_1\vec{v}_1 + k_2\vec{v}_2 + k_3\vec{v}_3 = \vec{0}$. Suppose one $k_i = 0$, this implies that $\vec{v}_1, \vec{v}_2$ or $\vec{v}_3$ is $\vec{0}$. But $\vec{v}_3 \notin \text{span}\{\vec{v}_1, \vec{v}_2\}$ $\therefore V_3 \neq 0$. But $\{\vec{v}_1, \vec{v}_2\}$ is linearly independent $\therefore \vec{v}_1 \neq 0 \neq \vec{v}_2$. $\lightning$

Suppose two $k_i = 0$.
$k_1 \neq 0 \neq k_2$ $\lightning$ since $\{\vec{v}_1, \vec{v}_2\}$ is linearly independent.
$k_2 \neq 0 \neq k_3$ $\lightning$ since $\vec{v}_3 \notin \text{span}\{\vec{v}_1, \vec{v}_2\}$
$k_1 \neq 0 \neq k_3$ $\lightning$ since $V_3 \notin \text{span}\{\vec{v}_1, \vec{v}_2\}$

Suppose three $k_i = 0$ $\lightning$ since $V_3 \notin \text{span}\{\vec{v}_1, \vec{v}_2\}$

$\therefore k_i = 0 \ \forall i$ $\therefore \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly independent.

Question 2. *(5 marks)* §4.4 #4d Is the following a basis for P_2?

$\{-4 + x + 3x^2, 6 + 5x + 2x^2, 8 + 4x + x^2\}$

① Is the set linearly independent? $k_1(-4 + x + 3x^2) + k_2(6 + 5x + 2x^2) + k_3(8 + 4x + x^2) = \vec{0}$

$$\begin{bmatrix} -4 & 6 & 8 \\ 1 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

k_i has only the trivial solution if $|A| \neq 0$

$$|A| = -4(5)(1) + 6(4)(3) + 8(1)(2) - 8(5)(3) - (-4)(4)(2) - 1(1)(6) = -20 + 72 + 16 - 120 + 32 - 6 = -26 \neq 0$$

② Does the set span P_2? Let $a + bx + cx^2 \in P_2$

$k_1(-4 + x + 3x^2) + k_2(6 + 5x + 2x^2) + k_3(8 + 4x + x^2) = a + bx + cx^2$

$$\begin{bmatrix} -4 & 6 & 8 \\ 1 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

since $|A| \neq 0$. a unique solution exists for any a, b, c.

$\therefore$ the set spans P_2

$\therefore$ the set is a basis for P_2.