Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S4: Fall 2014

Name: Y. Lamontagne
Student ID: ____________

Quiz 2

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §1.2 #25 *(4 marks)* Determine the values of a for which the system has no solutions, exactly one solution, or infinitely many solutions.

$$\begin{array}{rcrcrcl} x & + & 2y & - & 3z & = & 4 \\ 3x & - & y & + & 5z & = & 2 \\ 4x & + & y & + & (a^2-14)z & = & a+2 \end{array}$$

$$\begin{bmatrix} 1 & 2 & -3 & 4 \\ 3 & -1 & 5 & 2 \\ 4 & 1 & a^2-14 & a+2 \end{bmatrix}$$

$$\sim \begin{array}{r} \\ -3R_1+R_2 \to R_2 \\ -4R_1+R_3 \to R_3 \end{array} \begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & -7 & +14 & -10 \\ 0 & -7 & a^2-2 & a-14 \end{bmatrix}$$

$$\sim \begin{array}{r} \\ \\ -R_2+R_3 \to R_3 \end{array} \begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & 0 & a^2-16 & a-4 \end{bmatrix}$$

a) <u>no solutions:</u> the system needs a contradiction $0=b$ where $b \neq 0$: if $a=-4$

b) <u>unique solution:</u> the system needs #var. = # leading entries $a^2-16 \neq 0$: if $a \neq \pm 4$

c) <u>infinite # solution:</u> the system needs # var > # leading entries $a^2-16=0$ & $a-4=0$: if $a=4$

Question 3. §1.2 #20 *(6 marks)* Solve the given linear system by any method.

$$\begin{array}{rcrcrcrcl} & & v & + & 3w & - & 2x & = & 0 \\ 2u & + & v & - & 4w & + & 3x & = & 0 \\ 2u & + & 3v & + & 2w & - & x & = & 0 \\ -4u & - & 3v & + & 5w & - & 4x & = & 0 \end{array}$$

$$\begin{bmatrix} 0 & 1 & 3 & -2 & 0 \\ 2 & 1 & -4 & 3 & 0 \\ 2 & 3 & 2 & -1 & 0 \\ -4 & -3 & 5 & -4 & 0 \end{bmatrix} \sim \underset{R_1 \leftrightarrow R_2}{} \begin{bmatrix} 2 & 1 & -4 & 3 & 0 \\ 0 & 1 & 3 & -2 & 0 \\ 2 & 3 & 2 & -1 & 0 \\ -4 & -3 & 5 & -4 & 0 \end{bmatrix}$$

$$\sim \begin{array}{r} \\ \\ -R_1+R_3 \to R_3 \\ 2R_1+R_4 \to R_4 \end{array} \begin{bmatrix} 2 & 1 & -4 & 3 & 0 \\ 0 & 1 & 3 & -2 & 0 \\ 0 & 2 & 6 & -4 & 0 \\ 0 & -1 & -3 & 2 & 0 \end{bmatrix} \sim \begin{array}{r} \\ \\ -2R_2+R_3 \to R_3 \\ R_2+R_4 \to R_4 \end{array} \begin{bmatrix} 2 & 1 & -4 & 3 & 0 \\ 0 & 1 & 3 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{array}{r} -R_2+R_1 \to R_1 \\ \\ \\ \\ \end{array} \begin{bmatrix} 2 & 0 & -7 & 5 & 0 \\ 0 & 1 & 3 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{array}{r} \frac{1}{2}R_1 \to R_1 \\ \\ \\ \\ \end{array} \begin{bmatrix} 1 & 0 & -7/2 & 5/2 & 0 \\ 0 & 1 & 3 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Let $X_3 = s$, $X_4 = t$ $\quad s,t \in \mathbb{R}$

From augmented matrix

$$\begin{cases} X_1 - \frac{7}{2}X_3 + \frac{5}{2}X_4 = 0 \\ X_2 + 3X_3 - 2X_4 = 0 \end{cases}$$

$$X_1 = \frac{7}{2}s - \frac{5}{2}t$$
$$X_2 = -3s + 2t \qquad s,t \in \mathbb{R}$$
$$X_3 = s$$
$$X_4 = t$$