Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S4: Fall 2014

Name: Y. Lamontagne
Student ID:

Quiz 2

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §1.3 Consider the matrices

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix},\ B = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix},\ C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix},\ D = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix},\ E = \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}$$

In each part, compute the given expression (where possible).

#4h. *(2 marks)* $(2E^T - 3D^T)^T$

#4d. *(3 marks)* $\operatorname{tr}(C^TA^T + 2E^T)$

$$(2E^T - 3D^T)^T = \left(2\begin{bmatrix} 6 & -1 & 4 \\ 1 & 1 & 1 \\ 3 & 2 & 3 \end{bmatrix} - 3\begin{bmatrix} 1 & -1 & 3 \\ 5 & 0 & 2 \\ 2 & 1 & 4 \end{bmatrix}\right)^T$$

$$= \begin{bmatrix} 9 & 1 & -1 \\ -13 & 2 & -4 \\ 0 & 1 & -6 \end{bmatrix}^T = \begin{bmatrix} 9 & -13 & 0 \\ 1 & 2 & 1 \\ -1 & -4 & -6 \end{bmatrix}$$

Let $Y = C^TA^T + 2E^T = \begin{bmatrix} 1 & 3 \\ 4 & 1 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 3 & -1 & 1 \\ 0 & 2 & 1 \end{bmatrix} + 2\begin{bmatrix} 6 & -1 & 4 \\ 1 & 1 & 1 \\ 3 & 2 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 3 & 5 & 4 \\ 12 & -2 & 5 \\ 6 & 8 & 7 \end{bmatrix} + 2\begin{bmatrix} 6 & -1 & 4 \\ 1 & 1 & 1 \\ 3 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 15 & 3 & 12 \\ 14 & 0 & 7 \\ 12 & 12 & 13 \end{bmatrix}$$

then $\operatorname{tr}(Y) = 15 + 0 + 13 = 28$

Question 2. §1.4 #54 A square matrix A is said to be *idempotent* if $A^2 = A$.

1. a. *(2 marks)* Show that if A is idempotent, then so is $I - A$.
2. b. *(3 marks)* Show that if A is idempotent, then $2A - I$ is invertible and is its own inverse.

a) $$\begin{aligned}(I-A)^2 = (I-A)(I-A) &= I \cdot I - I \cdot A - A \cdot I + A^2 \\ &= I - A - A + A \quad \text{since } A^2 = A \\ &= I - A\end{aligned}$$

$\therefore$ $I-A$ is idempotent

b) $$\begin{aligned}(2A-I)(2A-I) &= 2A(2A) - 2A \cdot I - 2 \cdot I \cdot A + I \\ &= 4A^2 - 2A - 2A + I \\ &= 4A - 4A + I \quad \text{since } A^2 = A \\ &= I\end{aligned}$$

$\therefore$ $2A - I$ is invertible and its own inverse.