Dawson College: Linear Algebra (SCIENCE): 201-NYC-05-S4: Fall 2014

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Quiz ~~3~~ 4

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(2 marks)* §1.5 #7c.

$$A = \begin{bmatrix} 3 & 4 & 1 \\ 2 & -7 & -1 \\ 8 & 1 & 5 \end{bmatrix} \quad C = \begin{bmatrix} 3 & 4 & 1 \\ 2 & -7 & -1 \\ 2 & -7 & 3 \end{bmatrix}$$

Find an elementary matrix E that satisfies the equation.

$EA = C$

$$A \underset{-2R_1+R_3 \to R_3}{\sim} C$$

$$\text{So } I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \underset{-2R_1+R_3 \to R_3}{\sim} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} = E$$

Question 2. *(4 marks)* §1.5 #20 Use the inversion algorithm to find the inverse of the given matrix, if the inverse exists.

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ 1 & 3 & 5 & 0 \\ 1 & 3 & 5 & 7 \end{bmatrix}$$

$[A \mid I]$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 3 & 5 & 0 & 0 & 0 & 1 & 0 \\ 1 & 3 & 5 & 7 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \begin{array}{r} \\ -R_1+R_2 \to R_2 \\ -R_1+R_3 \to R_3 \\ -R_1+R_4 \to R_4 \end{array} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 3 & 5 & 0 & -1 & 0 & 1 & 0 \\ 0 & 3 & 5 & 7 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \begin{array}{r} \\ \frac{1}{3}R_2 \to R_2 \\ -R_2+R_3 \to R_3 \\ -R_2+R_4 \to R_4 \end{array} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 5 & 7 & 0 & -\cancel{1} & 0 & 1 \end{array}\right]$$

$$\begin{array}{r} \\ \\ \frac{1}{5}R_3 \to R_3 \\ -R_3+R_4 \to R_4 \end{array} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -\frac{1}{5} & \frac{1}{5} & 0 \\ 0 & 0 & 0 & 7 & 0 & 0 & -1 & 1 \end{array}\right]$$

$$\sim \begin{array}{r} \\ \\ \\ \frac{1}{7}R_4 \to R_4 \end{array} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -\frac{1}{5} & \frac{1}{5} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -\frac{1}{7} & \frac{1}{7} \end{array}\right]$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & -\frac{1}{5} & \frac{1}{5} & 0 \\ 0 & 0 & -\frac{1}{7} & \frac{1}{7} \end{bmatrix}$$

Question 3. *(4 marks)* §1.5 #20 Show that if

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & c \end{bmatrix}$$

is an elementary matrix, then at least one entry in the third row must be zero.

1. the elem. matrix is not obtained by changing the order of two rows since the first two rows are identical to the identity.
2. if the elem. matrix is obtained by mult row 3 by a constant then row 3 is of the form $0 \; 0 \; c$ $\therefore a=b=0$
3. a) if the elem. matrix is obtained by adding a mult. of row 2 to row 3 then row 3 is of the form $0 \; b \; 1$ $\therefore a=0$
   b) if the elem. matrix is obtained by adding a mult. of row 1 to row 3 then row 3 is of the form $a \; 0 \; 1$ $\therefore b=0$.