Dawson College: Linear Algebra (SCIENCE): 201-NYC-05-S4: Fall 2014

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Quiz 5

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §1.6 #18a Consider the matrices

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 2 & 2 & -2 \\ 3 & 1 & 1 \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Show that the equation $A\mathbf{x} = \mathbf{x}$ can be rewriten as $(A - I)\mathbf{x} = \mathbf{0}$ and use this result to solve $A\mathbf{x} = \mathbf{x}$.

$$(A - I)x = 0$$
$$Ax - Ix = 0$$
$$Ax - x = 0$$
$$Ax = x$$

So

$$A - I = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & -2 \\ 3 & 1 & 0 \end{bmatrix}$$

and

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 2 & 1 & -2 & 0 \\ 3 & 1 & 0 & 0 \end{array}\right] \sim \begin{array}{l} \\ -2R_1 + R_2 \to R_2 \\ -3R_1 + R_3 \to R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & -1 & -6 & 0 \\ 0 & -2 & -6 & 0 \end{array}\right]$$

$$\sim \begin{array}{l} \\ \\ -2R_2 + R_3 \to R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & -1 & -6 & 0 \\ 0 & 0 & 6 & 0 \end{array}\right]$$

unique solution since # leading val in var column = # var.
$\therefore$ trivial sol. since homogeneous system.

$$x = 0$$

Question 2. *(2 marks)* §1.7 #28 Find a diagonal matrix A that satisfies the given condition:

$$A^{-2} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

invertible since all elements of main diagonal are nonzero

$$A^2 = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \longrightarrow A^2 = \begin{bmatrix} 1/9 & 0 & 0 \\ 0 & 1/4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} \pm 1/3 & 0 & 0 \\ 0 & \pm 1/2 & 0 \\ 0 & 0 & \pm 1 \end{bmatrix}$$

Question 3. *(3 marks)* §1.7 #33 Prove: If $A^T A = A$, then A is symmetric and $A = A^2$.

$$A^T = (A^T A)^T$$
$$= (A^T (A^T)^T)$$
$$= A^T A$$

$\therefore A$ is symmetric

$$A^2 = AA$$
$$= A^T A \quad \text{since } A \text{ is symmetric}$$
$$= A$$