Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S4: Fall 2014

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Quiz 6

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §2.1 #34 Show that the matrices

$$A = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$$

and

$$B = \begin{bmatrix} d & e \\ 0 & f \end{bmatrix}$$

commute if and only if

$$\begin{vmatrix} b & a-c \\ e & d-f \end{vmatrix} = 0$$

[=>] Premise A & B commute then $ae+bf = bd+ce$ from *

So $\begin{vmatrix} b & a-c \\ e & d-f \end{vmatrix} = 0$ by **

[<=] Premise $\begin{vmatrix} b & a-c \\ e & d-f \end{vmatrix} = 0$ then $bd+ce = bf+ae$ from **

So A & B commute by *

So

$AB = BA$

$$\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}\begin{bmatrix} d & e \\ 0 & f \end{bmatrix} = \begin{bmatrix} d & e \\ 0 & f \end{bmatrix}\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$$

$$* \quad \begin{bmatrix} ad & ae+bf \\ 0 & cf \end{bmatrix} = \begin{bmatrix} ad & bd+ce \\ 0 & cf \end{bmatrix}$$

and $\begin{vmatrix} b & a-c \\ e & d-f \end{vmatrix} = 0$

$$** \quad 0 = b(d-f) - e(a-c)$$
$$0 = bd - bf - ea + ce$$
$$bd + ce = bf + ae$$

Question 2. *(5 marks)* §2.2 #19 Decide whether the given matrix is invertible, and if so, use the adjoint method to find its inverse.

$$A = \begin{bmatrix} 2 & 5 & 5 \\ -1 & -1 & 0 \\ 2 & 4 & 3 \end{bmatrix} \begin{matrix} 2 & 5 \\ -1 & -1 \\ 2 & 4 \end{matrix}$$

$$A^{-1} = \frac{1}{\det A} \operatorname{adj} A$$

$$|A| = 2(-1)(3) + 5(-1)(4) - 5(-1)(2) - 5(-1)(3)$$
$$= -6 - 20 + 10 + 15$$
$$= -1$$

$$C_{11} = \begin{vmatrix} -1 & 0 \\ 4 & 3 \end{vmatrix} = -3$$

$$C_{12} = (-1)\begin{vmatrix} -1 & 0 \\ 2 & 3 \end{vmatrix} = 3$$

$$C_{13} = \begin{vmatrix} -1 & -1 \\ 2 & 4 \end{vmatrix} = -2$$

$$C_{21} = -\begin{vmatrix} 5 & 5 \\ 4 & 3 \end{vmatrix} = 5$$

$$C_{22} = \begin{vmatrix} 2 & 5 \\ 2 & 3 \end{vmatrix} = -4$$

$$C_{23} = (-1)\begin{vmatrix} 2 & 5 \\ 2 & 4 \end{vmatrix} = 2$$

$$C_{31} = \begin{vmatrix} 5 & 5 \\ -1 & 0 \end{vmatrix} = 5$$

$$C_{32} = (-1)\begin{vmatrix} 2 & 5 \\ -1 & 0 \end{vmatrix} = 5$$

$$C_{33} = \begin{vmatrix} 2 & 5 \\ -1 & -1 \end{vmatrix} = 3$$

$$\operatorname{adj} A = [\text{cofactor matrix}]^T$$

$$= \begin{bmatrix} -3 & 3 & -2 \\ 5 & -4 & 2 \\ 5 & 5 & 3 \end{bmatrix}^T$$

$$= \begin{bmatrix} -3 & 5 & 5 \\ 3 & -4 & 5 \\ -2 & 2 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-1}\begin{bmatrix} -3 & 5 & 5 \\ 3 & -4 & 5 \\ -2 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 3 & -5 & -5 \\ -3 & 4 & -5 \\ 2 & -2 & -3 \end{bmatrix}$$