Dawson College: Linear Algebra (SCIENCE): 201-NYC-05-S4: Fall 2014

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Quiz 11

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §4.1 #1 Let V be the set of all ordered pairs of real numbers, and consider the following addition and scalar multiplication operations on $\vec{u} = (u_1, u_2)$ and $\vec{v} = (v_1, v_2)$:

$$\vec{u} + \vec{v} = (u_1 + v_1, u_2 + v_2) \text{ and } k\vec{u} = (0, ku_2)$$

a. Compute $\vec{u}+\vec{v}$ and $k\vec{u}$ for $\vec{u} = (-1,2)$, $\vec{v} = (3,4)$, and $k = 3$.

b. In words, explain why V is closed under addition and scalar multiplication.

c. Since addition on V is the standard addition operation on $\mathbb{R}^2$, certain vector space axioms hold for V because they are known to hold for $\mathbb{R}^2$. Which axioms are they?

d. Show that Axioms 7, 8, and 9 hold.

e. Show that Axiom 10 fails and hence that V is not a vector space under the given operations.

a) $\vec{u}+\vec{v} = (2,6)$, $K\vec{u} = 3(-1,2) = (0,6)$

b) addition and scalar multiplication gives an ordered pair.

c) 1, 2, 3, 4, 5

e) $1\cdot\vec{u} = (0, 1\cdot u_2)$
$= (0, u_2) \neq \vec{u}$

d) 7 $K(\vec{u}+\vec{v}) = K\vec{u} + K\vec{v}$

$$\begin{aligned} LHS &= K(\vec{u}+\vec{v}) \\ &= K((u_1,u_2)+(v_1,v_2)) \\ &= K\ (u_1+v_1, u_2+v_2) \\ &= (0, K(u_2+v_2)) \\ &= (\ , Ku_2+Kv_2) \end{aligned}$$

$$\begin{aligned} RHS &= K\vec{u} + K\vec{v} \\ &= (0,Ku_2) + (0,Kv_2) \\ &= (0, Ku_2+Kv_2) \end{aligned}$$

8 $(K+m)\vec{u} = K\vec{u} + m\vec{u}$

$$\begin{aligned} LHS &= (0, (K+m)u_2) \\ &= (0, Ku_2+mu_2) \end{aligned}$$

$$\begin{aligned} RHS &= (0,Ku_2) + (0,mu_2) \\ &= (0, Ku_2+mu_2) \end{aligned}$$

9 $K(m\vec{u}) = (Km)\vec{u}$

$$\begin{aligned} LHS &= K(m(u_1,u_2)) \\ &= K((0,mu_2)) \\ &= (0, Kmu_2) \end{aligned}$$

$$\begin{aligned} RHS &= (Km)(u_1,u_2) \\ &= (0, Kmu_2) \end{aligned}$$

Question 2. *(5 marks)* §4.2 #2c Use the subspace test to determine which of the following are subspaces of M_{nn}.

The set of all $n \times n$ matrices A such that $\text{tr}(A) = 0$. $W = \{A \mid A \in M_{nn} \text{ and } trA = 0\}$

Lets apply the subspace test.

a) Let $A = [a_{ij}]_{n\times n}$, $B = [b_{ij}]_{n\times n} \in W$ then $A+B = [a_{ij}+b_{ij}]_{n\times n} \in W$

since
$$\begin{aligned} tr(A+B) &= (a_{11}+b_{11}) + (a_{22}+b_{22}) + \cdots + (a_{nn}+b_{nn}) \\ &= a_{11}+a_{22}+\cdots+a_{nn} + b_{11}+b_{22}+\cdots+b_{nn} \\ &= trA + trB = 0+0 \quad \text{since } A,B \in W \end{aligned}$$

b) $K \in \mathbb{R}$ and $A = [a_{ij}]_{n\times n} \in W$ then $KA \in W$

since
$$\begin{aligned} tr(KA) &= Ka_{11} + Ka_{22} + \cdots + Ka_{nn} = K(a_{11}+a_{22}+\cdots+a_{nn}) \\ &= K\,trA = K0 = 0 \end{aligned}$$

since $A \in W$