Name: ____________________

Student ID: ____________________

Test 1

This test is graded out of 45 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1.

a. *(6 marks)* Solve the following system by Gauss-Jordan elimination:

$$\begin{array}{rcrcrcrcrcr} 3x_1 & - & 2x_2 & + & x_3 & - & 3x_4 & + & x_5 & = & 0 \\ 4x_1 & + & 3x_2 & - & x_3 & + & x_4 & - & 2x_5 & = & 0 \\ 7x_1 & + & x_2 & & & - & 2x_4 & - & x_5 & = & 0 \\ & & & & x_3 & - & x_4 & & & = & 0 \end{array}$$

b. *(2 marks)* Find two particular nontrivial solution to the above system.

Question 2. Consider the matrices:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & 2 & 1 \\ 0 & -4 & 0 \end{bmatrix} C = \begin{bmatrix} 0 & 1 \\ -3 & 0 \\ 3 & -1 \end{bmatrix} D = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix} F = \begin{bmatrix} 1 & -1 \\ 0 & -2 \\ 1 & -3 \end{bmatrix}$$

a. *(2 marks)* Compute the following, if possible.

$$\operatorname{tr}(BC)$$

b. *(2 marks)* Compute the following, if possible.

$$A^{-2}$$

c. *(6 marks)* Find E, if possible.

$$(\operatorname{tr}(D)D - E^t)^{-1} = BA^{-2}F$$

Question 3. Consider

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{bmatrix}.$$

a. *(4 marks)* Find A^{-1}.

b. *(2 marks)* Write A^{-1} as a product of elementary matrices.

c. *(2 marks)* Using a. solve $Ax = b$ where $x = [x_1, x_2, x_3]^t$ and $b = [1, -2, 1]^t$.

Question 4.[1]

a. *(2 marks)* Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$. Find a 2×2 matrix B such that $AB = \mathbf{0}$ but $BA \neq \mathbf{0}$.

b. Let A and B be non-zero $n \times n$ matrices such that $AB = \mathbf{0}$ but $BA \neq \mathbf{0}$.

   i. *(2 marks)* Prove that $(BA)^2 = \mathbf{0}$

   ii. *(2 marks)* Prove that B is singular.

Question 5.[2] *(4 marks)* Let

$$A = \begin{bmatrix} 2 & 1 \\ x & y \end{bmatrix}$$

Find x and y such that A^2 is symmetric, if possible.

[1]From a John Abbott Final Examination

[2]From a John Abbott Final Examination

Question 6.[3] A matrix X is called a *weak generalized inverse* of A if $AXA = A$

a. *(3 marks)* For what value of k is $\begin{bmatrix} k & k \\ k & k \\ k & k \end{bmatrix}$ a weak generalized inverse of $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

b. *(2 marks)* Show that if the system $A\mathbf{x} = \mathbf{b}$ is consistent then $X\mathbf{b}$ will be a solution to this system

[3] From a John Abbott Final Examination

Question 7.*(6 marks)* Consider the following systems:

$$\begin{array}{rcrcrcl} x & + & y & + & 7z & = & c_1 \\ x & + & 2y & + & 3z & = & c_2 \\ 2x & + & 3y & + & 10z & = & 1 \end{array}$$

and

$$\begin{array}{rcrcrcl} 2x & + & y & - & 7z & = & c_1 \\ 3x & - & 2y & + & 3z & = & c_2 \\ 7x & & & - & 11z & = & 0 \end{array} .$$

If any, for what value(s) of c_1, c_2 are both systems consistent.

Bonus Question. *(5 marks)* If A is 2×2, show that $A^{-1} = A^T$ if and only if $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ for some θ or $A = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$ for some θ.