Dawson College: **Linear Algebra**: 201-NYC-05-S04: Fall 2014

Name: ____________________
Student ID: ____________________

Test 3

This test is graded out of 40 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Let $\mathscr{H} = \{A|\, A \text{ is a } 2\times 2 \text{ matrix and } A^T = -A\}$ with the usual addition and scalar multiplication.

a. *(2 marks)* Give an example of a non-zero matrix in $\mathscr{H}$. Justify.

b. *(2 marks)* Does $\mathscr{H}$ satisfy closure under vector addition? Justify.

c. *(2 marks)* Does $\mathscr{H}$ contain the zero vector of $\mathscr{M}_{2\times 2}$ (the vector space of 2×2 matrices)? Justify.

d. *(2 marks)* Does $\mathscr{H}$ satisfy closure under scalar multiplication? Justify.

e. *(2 marks)* Is $\mathscr{H}$ a vector subspace of $\mathscr{M}_{2\times 2}$ (the vector space of 2×2 matrices)? Justify.

Question 2. *(2 marks)* Determine whether the following is a vector space:

$$\mathscr{Y} = \{(1,y) \mid y \in \mathbb{R}\}$$

with the following polynomial addition and scalar multiplication.

$$(1,y_1)+(1,y_2) = (1,y_1+y_2)$$

and

$$k(1,y) = (1,y)$$

Question 3. Let $\mathscr{W} = \{p(x) = a_0 + a_1x + a_2x^2 \mid p(1) = 0\}$ be a vector subspace of $\mathscr{P}_n$.

a. *(4 marks)* Find a basis S for $\mathscr{W}$.

b. *(2 marks)* Determine the dimension of $\mathscr{W}$, Justify.

c. *(2 marks)* Find the coordinate vector of $p(x) = -1 - x + 2x^2$ relative to the basis S.

Question 4. Let $\vec{u} = (1,0,2)$ and $\vec{v} = (1,-2,3)$.

a. *(2 marks)* Find a vector $\vec{w}$ of length $\sqrt{21}$ orthogonal to $\vec{u}$ and $\vec{v}$

b. *(2 marks)* Compute the scalar triple product of $\vec{u}$, $\vec{v}$ and $\vec{w}$

c. *(2 marks)* Find the volume of the parallelepiped with sides $3\vec{u}, 2\vec{v}$ and $-\vec{w}$.

d. *(4 marks)* Let $\mathscr{P}$ be a plane that passes through the point $P_0(2,1,-1)$ and is parallel to $\vec{u}$ and $\vec{v}$. Find the point on $\mathscr{P}$ closest to $P(3,2,1)$.

Question 5. [1] Let $\vec{u}_1 = (\lambda, \lambda, 2)$, $\vec{u}_2 = (\lambda, 2, \lambda)$ and $\vec{u}_3 = (1, \lambda, -\lambda)$

a. *(5 marks)* For what value(s) of λ if any, is $\{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$ linearly independent? If $\lambda = 1$ is one of those values, express $(1,2,3)$ as a linear combibation of $\{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$ when $\lambda = 1$.

b. *(3 marks)* For what value(s) of λ if any, is $\{\vec{u}_2, \vec{u}_3\}$ linearly dependent? What geometrical object is span$\{\vec{u}_2, \vec{u}_3\}$?

c. *(2 marks)* For what value(s) of λ if any, is span$\{\vec{u}_1, \vec{u}_2\}$ a line in $\mathbb{R}^3$.

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[1] Modified from a John Abbott Final Examination

Bonus Question. *(5 marks)* If U and W are subspaces of a vector space V then

$$U \cap W = \left\{ \vec{v} \,\middle|\, \vec{v} \in U \text{ and } \vec{v} \in W \right\}.$$

is a subspace of V.

Show that $U \cap W = \{\vec{0}\}$ if and only if $\{\vec{u}, \vec{v}\}$ is linearly independent for any nonzero vectors $\vec{u} \in U$ and $\vec{v} \in W$.