Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S5: Fall 2014

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Quiz 10

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §3.4 #19 Find the general solution to the linear system and confirm that the row vectors of the coefficient matrix are orthogonal to the solution vector.

$$\begin{aligned} x_1+5x_2+x_3+2x_4-x_5 &= 0 \\ x_1-2x_2-x_3+3x_4+2x_5 &= 0 \end{aligned}$$

$$\begin{bmatrix} 1 & 5 & 1 & 2 & -1 & 0 \\ 1 & -2 & -1 & 3 & 2 & 0 \end{bmatrix}$$

$$\sim_{-R_1+R_2\to R_2} \begin{bmatrix} 1 & 5 & 1 & 2 & -1 & 0 \\ 0 & -7 & -2 & 1 & 3 & 0 \end{bmatrix}$$

$$\sim_{-\frac{1}{7}R_2} \begin{bmatrix} 1 & 5 & 1 & 2 & -1 & 0 \\ 0 & 1 & 2/7 & -1/7 & -3/7 & 0 \end{bmatrix}$$

$$\sim_{-5R_2+R_1\to R_1} \begin{bmatrix} 1 & 0 & -3/7 & 19/7 & 8/7 & 0 \\ 0 & 1 & 2/7 & -1/7 & -3/7 & 0 \end{bmatrix}$$

$x_3=s$, $x_4=t$, $x_5=r$

$x_1 = \frac{3}{7}s - \frac{19}{7}t - \frac{8}{7}r$ $\quad x_2 = -\frac{2}{7}s + \frac{1}{7}t + \frac{3}{7}r$

Let $\vec{r}_1 = (1,5,1,2,-1)$, $\vec{r}_2 = (1,-2,-1,3,2)$

$\vec{x} = \left(\frac{3}{7}s - \frac{19}{7}t - \frac{8}{7}r, -\frac{2}{7}s + \frac{1}{7}t + \frac{3}{7}r, s, t, r\right)$

$$\vec{x}\cdot\vec{r}_1 = \frac{3}{7}s - \frac{19}{7}t - \frac{8}{7}r + 5\left(-\frac{2}{7}s + \frac{1}{7}t + \frac{3}{7}r\right) + 1\cdot s + 2\cdot t + (-1)r = 0$$

$$\vec{x}\cdot\vec{r}_2 = \frac{3}{7}s - \frac{19}{7}t - \frac{8}{7}r + (-2)\left(-\frac{2}{7}s + \frac{1}{7}t + \frac{8}{7}r\right) + (-1)(s) + 3(t) + 2(r) = 0$$

Question 2. *(5 marks)* §3.4 #21

a. The equation $x+y+z=1$ can be viewed as a linear system of one equation in three unknowns. Express a general solution of this equation as a particular solution plus a general solution of the associated homogeneous system.

b. Give a geometric interpretation of the result in part a..

a) a particular solution of $x+y+z=1$ is $(1,0,0)$
the general solution of $x+y+z=0$, let $y=s$, $z=t$, $x=-s-t$,
so $(x,y,z) = s(-1,1,0) + t(-1,0,1)$

$\therefore$ the general solution of $x+y+z=1$ is

$$(x,y,z) = (1,0,0) + s\underbrace{(-1,1,0)}_{\vec{d}_1} + t\underbrace{(-1,0,1)}_{\vec{d}_2}$$

b)