Name: Y. Lamontagne
Student ID: _____

Quiz 11

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §4.1 #2 Let V be the set of all ordered pairs of real numbers, and consider the following addition and scalar multiplication operations on $\vec{u} = (u_1, u_2)$ and $\vec{v} = (v_1, v_2)$.

$$\vec{u} + \vec{v} = (u_1 + v_1 + 1, u_2 + v_2 + 1) \qquad k\vec{u} = (ku_1, ku_2)$$

a. Compute $\vec{u} + \vec{v}$ and $k\vec{u}$ for $\vec{u} = (0,4)$ and $\vec{v} = (1,-3)$, and $k = 2$.

a) $\vec{u} + \vec{v} = (0+1+1,\ 4-3+1) = (2,2)$

$Ku = (2\cdot 0,\ 2\cdot 4) = (0,8)$

b. Show that $(0,0) \neq \vec{0}$.

b) $(0,0) + (u_1, u_2) = (u_1+1,\ u_2+1) \neq (u_1, u_2)$

c. Show that $(-1,-1) = \vec{0}$.

d. Show that Axiom 5 holds by producing an ordered pair $-\vec{u}$ such that $\vec{u} + (-\vec{u}) = \vec{0}$ for $\vec{u} = (u_1, u_2)$.

e. Find two vector space axioms that fail to hold.

c) $\vec{0} + \vec{u} = (-1,-1) + (u_1, u_2) = (-1+u_1+1,\ -1+u_2+1) = (u_1, u_2) = \vec{u} \quad \forall \vec{u} \in V$

$\therefore \vec{0} = (-1,-1)$

d) $\vec{u} + \vec{w} = (u_1, u_2) + (w_1, w_2)$

$= (u_1 + w_1 + 1,\ u_2 + w_2 + 1) = \vec{0} = (-1,-1)$

$u_1 + w_1 + 1 = -1 \qquad w_1 = -2 - u_1$

$u_2 + w_2 + 1 = -1 \qquad w_2 = -2 - u_2$

$-\vec{u} = \vec{w} = (-2-w_1,\ -2-w_2)$

e) $(s+r)\vec{u} = s\vec{u} + r\vec{u}$ fails

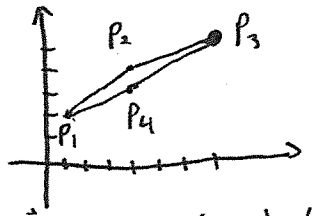
i.e. $((s+r)u_1,\ (s+r)u_2) \neq ((s+r)u_1 + 1,\ (s+r)u_2 + 1)$

$r(\vec{u} + \vec{v}) = r\vec{u} + r\vec{v}$ fails

$(r(u_1+v_1+1),\ r(u_2+v_2+1)) \neq (r(u_1+v_1)+1,\ r(u_2+v_2)+1)$

Question 2. *(3 marks)* §3.5 #11 Find the area of the parallelogram with the given vertices.

$P_1(1,2)$, $P_2(4,4)$, $P_3(7,5)$ $P_4(4,3)$



$\vec{P_1P_2} = P_2 - P_1 = (4,4) - (1,2) = (3,2)$

$\vec{P_1P_4} = P_4 - P_1 = (4,3) - (1,2) = (3,1)$

Area $= \left|\begin{vmatrix} 3 & 2 \\ 3 & 1 \end{vmatrix}\right|$

$= \left|(3)(1) - 2(3)\right| = |-3| = 3$

Question 3. *(2 marks)* §3.5 #15 Find the area of the triangle in 3-space that has the given vertices

$P_1(2,6,-1)$, $P_2(1,1,1)$, $P_3(4,6,2)$

$\vec{P_1P_2} = P_2 - P_1 = (1,1,1) - (2,6,-1) = (-1,-5,2)$

$\vec{P_1P_3} = P_3 - P_1 = (4,6,2) - (2,6,-1) = (2,0,3)$

$\vec{P_1P_2} \times \vec{P_1P_3} = \begin{vmatrix} -1 & 2 \\ -5 & 0 \\ 2 & 3 \end{vmatrix} = \left(\begin{vmatrix} -5 & 0 \\ 2 & 3 \end{vmatrix}, -\begin{vmatrix} -1 & 2 \\ 2 & 3 \end{vmatrix}, \begin{vmatrix} -1 & 2 \\ -5 & 0 \end{vmatrix}\right) = (-15,\ 7,\ +10)$

Area $= \dfrac{\|\vec{P_1P_2} \times \vec{P_1P_3}\|}{2} = \dfrac{\sqrt{(-15)^2 + 7^2 + (+10)^2}}{2} = \dfrac{\sqrt{374}}{2}$