Dawson College: Linear Algebra (SCIENCE): 201-NYC-05-S5: Fall 2014

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Quiz 12

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §4.3 #2 Prove: For any vectors $\vec{u}$, $\vec{v}$, and $\vec{w}$ in a vector space V, the vectors $\vec{u}-\vec{v}$, $\vec{v}-\vec{w}$, and $\vec{w}-\vec{u}$ form a linearly dependent set.

$$K_1(\vec{u}-\vec{v}) + K_2(\vec{v}-\vec{w}) + K_3(\vec{w}-\vec{u}) = \vec{0}$$

the set is linearly independent iff $K_i = 0\ \forall i$ is the only solution.

$$(K_1-K_3)\vec{u} + (K_2-K_1)\vec{v} + (K_3-K_2)\vec{w} = \vec{0}$$

since $\vec{u}, \vec{v}, \vec{w}$ are linearly independent.

$$\begin{aligned} K_1 - K_3 &= 0 \\ K_2 - K_1 &= 0 \\ K_3 - K_2 &= 0 \end{aligned} \qquad \underbrace{\begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}}_{A} \begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{vmatrix} \begin{matrix} 1 & 0 \\ -1 & 1 \\ 0 & -1 \end{matrix} = 1 + 0 - 1 - 0 - 0 - 0 = 0$$

$\therefore$ not only the trivial solution

$\therefore$ linearly dependent set.

Question 2. *(5 marks)* §4.4 #15 Find the coordinate vector of $\vec{p} = 2 - x + x^2$ relative to the basis $S = \{1+x,\ 1+x^2,\ x+x^2\}$.

$$\vec{p} = c_1 p_1 + c_2 p_2 + c_3 p_3 \qquad \therefore (\vec{p})_S = (0, 2, -1)$$

$$2 - x + x^2 = c_1(1+x) + c_2(1+x^2) + c_3(x+x^2)$$

$$2 - x + x^2 = (c_1+c_2)\cdot 1 + (c_1+c_3)x + (c_2+c_3)x^2$$

$$\begin{aligned} c_1 + c_2 &= 2 \\ c_1 + c_3 &= -1 \\ c_2 + c_3 &= 1 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 1 & 0 & 1 & -1 \\ 0 & 1 & 1 & 1 \end{array}\right] \underset{-R_1+R_2\to R_2}{\sim} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & -1 & 1 & -3 \\ 0 & 1 & 1 & 1 \end{array}\right] \underset{R_2+R_3\to R_3}{\sim} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & -1 & 1 & -3 \\ 0 & 0 & 2 & -2 \end{array}\right]$$

$$\underset{\frac{1}{2}R_3\to R_3}{\sim} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & -1 & 1 & -3 \\ 0 & 0 & 1 & -1 \end{array}\right] \underset{-R_3+R_2\to R_2}{\sim} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & -1 & 0 & -2 \\ 0 & 0 & 1 & -1 \end{array}\right] \underset{\substack{R_2+R_1\to R_1 \\ -R_2\to R_2}}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{array}\right]$$