

Quiz 2

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §1.2 #2d (2 marks) Determine whether the matrix is in row echelon form, reduced row echelon form, both, or neither.

$$\begin{bmatrix} 1 & 5 & -3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Row echelon form since

- row of zero is at bottom
- leading 1 to the right of the one above
- leading entry of each row is a 1.

Not Reduced row echelon form since 5 above the 2nd leading 1.

Question 2. §1.2 #3d (2 marks) Suppose that the augmented matrix for a system of linear equations has been reduced by row operations to the given row echelon form. Solve the system.

$$\begin{bmatrix} 1 & -3 & 7 & 1 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

From the 3rd row $0x_1 + 0x_2 + 0x_3 = 1$
 $0 = 1$



the system has no solution

∴ the system is inconsistent

Question 3. §1.2 #23 (6 marks) Solve the given linear system by any method.

$$2I_1 - I_2 + 3I_3 + 4I_4 = 9$$

$$I_1 - 2I_3 + 7I_4 = 11$$

$$3I_1 - 3I_2 + I_3 + 5I_4 = 8$$

$$2I_1 + I_2 + 4I_3 + 4I_4 = 10$$

$$\begin{bmatrix} 2 & -1 & 3 & 4 & 9 \\ 1 & 0 & -2 & 7 & 11 \\ 3 & -3 & 1 & 5 & 8 \\ 2 & 1 & 4 & 4 & 10 \end{bmatrix}$$

$$\sim R_1 \leftrightarrow R_2 \begin{bmatrix} 1 & 0 & -2 & 7 & 11 \\ 2 & -1 & 3 & 4 & 9 \\ 3 & -3 & 1 & 5 & 8 \\ 2 & 1 & 4 & 4 & 10 \end{bmatrix}$$

$$\sim \begin{array}{l} -2R_1 + R_2 \rightarrow R_2 \\ -3R_1 + R_3 \rightarrow R_3 \\ -2R_1 + R_4 \rightarrow R_4 \end{array} \begin{bmatrix} 1 & 0 & -2 & 7 & 11 \\ 0 & -1 & 7 & -10 & -13 \\ 0 & -3 & 7 & -16 & -25 \\ 0 & 1 & 8 & -10 & -12 \end{bmatrix}$$

$$\sim \begin{array}{l} -3R_2 + R_3 \rightarrow R_3 \\ R_2 + R_4 \rightarrow R_4 \end{array} \begin{bmatrix} 1 & 0 & -2 & 7 & 11 \\ 0 & -1 & 7 & -10 & -13 \\ 0 & 0 & -14 & 14 & 14 \\ 0 & 0 & 15 & -20 & -25 \end{bmatrix}$$

$$\sim \begin{array}{l} -\frac{1}{14} R_3 \end{array} \begin{bmatrix} 1 & 0 & -2 & 7 & 11 \\ 0 & -1 & 7 & -10 & -13 \\ 0 & 0 & 1 & -1 & -1 \\ 0 & 0 & 15 & -20 & -25 \end{bmatrix}$$

$$\sim \begin{array}{l} -15R_3 + R_4 \rightarrow R_4 \end{array} \begin{bmatrix} 1 & 0 & -2 & 7 & 11 \\ 0 & -1 & 7 & -10 & -13 \\ 0 & 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & -5 & -10 \end{bmatrix}$$

$$\sim -R_2 \begin{bmatrix} 1 & 0 & -2 & 7 & 11 \\ 0 & 1 & -7 & 10 & 13 \\ 0 & 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\sim \begin{array}{l} -7R_4 + R_1 \rightarrow R_1 \\ -10R_4 + R_2 \rightarrow R_2 \\ R_4 + R_3 \rightarrow R_3 \end{array} \begin{bmatrix} 1 & 0 & -2 & 0 & -3 \\ 0 & 1 & -7 & 0 & -7 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\sim \begin{array}{l} 2R_1 + R_1 \rightarrow R_1 \\ 7R_3 + R_2 \rightarrow R_2 \end{array} \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\therefore (I_1, I_2, I_3, I_4) = (-1, 0, 1, 2)$$