Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S5: Fall 2014

Name: Y. Lamontagne
Student ID:

Quiz 3

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §1.3 #21 *(3 marks)* Prove: If A and B are $n \times n$ matrices, then $\mathrm{tr}(A+B) = \mathrm{tr}(A) + \mathrm{tr}(B)$.

Let $A = [a_{ij}]_{n\times n}$, $B = [b_{ij}]_{n\times n}$ then $A + B = [a_{ij} + b_{ij}]_{n\times n}$

$tr(A) = a_{11} + a_{22} + \dots + a_{nn}$

$tr(B) = b_{11} + b_{22} + \dots + b_{nn}$

and $tr(A+B) = a_{11} + b_{11} + a_{22} + b_{22} + \dots + a_{nn} + b_{nn}$

$= a_{11} + a_{22} + \dots + a_{nn} + b_{11} + b_{22} + \dots + b_{nn}$

$\therefore tr(A+B) = tr(A) + tr(B)$

Question 2. §1.3 #30a *(3 marks)* Let $\mathbf{0}$ denote a 2×2 matrix, each of whose entries is zero. Is there a 2×2 matrix A such that $A \neq \mathbf{0}$ and $AA = \mathbf{0}$? Justify your answer.

yes,

$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \neq 0$ and $AA = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Question 3. §1.3 Consider the matrices

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}, C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}, D = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}, E = \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}$$

In each part, compute the given expression (where possible).

#3j. *(2 marks)* $B^T + 5C^T$

#4d. *(2 marks)* $(DA)^T$

$B^T + 5C^T = \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix} + 5\begin{bmatrix} 1 & 3 \\ 4 & 1 \\ 2 & 5 \end{bmatrix}$ not defined since matrix dimension are different.

$DA = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 12 \\ -2 & 1 \\ 11 & 8 \end{bmatrix}$

So $(DA)^T = \begin{bmatrix} 0 & -2 & 11 \\ 12 & 1 & 8 \end{bmatrix}$