

Quiz 4

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §1.4 #17 (5 marks) Use the given information to find A .

$$(I+2A)^{-1} = \begin{bmatrix} -1 & 2 \\ 4 & 5 \end{bmatrix}$$

$$((I+2A)^{-1})^{-1} = \begin{bmatrix} -1 & 2 \\ 4 & 5 \end{bmatrix}^{-1}$$

$$I+2A = \frac{1}{-13} \begin{bmatrix} 5 & -2 \\ -4 & -1 \end{bmatrix}$$

$$2A = \begin{bmatrix} -5/13 & 2/13 \\ 4/13 & 1/13 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$2A = \begin{bmatrix} -18/13 & 2/13 \\ 4/13 & -12/13 \end{bmatrix}$$

$$A = \begin{bmatrix} -9/13 & 1/13 \\ 2/13 & -6/13 \end{bmatrix}$$

Question 2. §1.4 #28 (3 marks) Show that if a square matrix A satisfies $A^2 - 3A + I = 0$, then $A^{-1} = 3I - A$

$$A^2 - 3A + I = 0$$

$$I = 3A - A^2 \quad \text{and} \quad I = (3I - A)A$$

$$I = A(3I - A)$$

∴ A is invertible and $A^{-1} = 3I - A$

Question 3. §1.3 #30 (2 marks) Assuming that all matrices are $n \times n$ and invertible, solve for D

$$ABC^T DBA^T C = AB^T$$

$$(ABC^T)^{-1} ABC^T DBA^T C (BA^T C)^{-1} = (ABC^T)^{-1} AB^T (BA^T C)^{-1}$$

$$I D I = (C^T)^{-1} B^{-1} A^{-1} A B^T C^{-1} (A^T)^{-1} B^{-1}$$

$$D = (C^T)^{-1} B^{-1} B^T C^{-1} (A^T)^{-1} B^{-1}$$