

Quiz 5

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §1.5 #29 (5 marks) Write the given matrix as a product of elementary matrices

$$A = \begin{bmatrix} -3 & 1 \\ 2 & 2 \end{bmatrix} \sim 3R_2 \rightarrow R_2 \begin{bmatrix} -3 & 1 \\ 6 & 6 \end{bmatrix} \sim 2R_1 + R_2 \rightarrow R_2 \begin{bmatrix} -3 & 1 \\ 0 & 8 \end{bmatrix} \sim \frac{1}{8}R_2 \rightarrow R_2 \begin{bmatrix} -3 & 1 \\ 0 & 1 \end{bmatrix}$$

where

$$E_1^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{3} \end{bmatrix}$$

$$E_6^{-1} = \begin{bmatrix} -3 & 0 \\ 0 & 1 \end{bmatrix}$$

$$E_2^{-1} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

$$E_3^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 8 \end{bmatrix}$$

$$E_4^{-1} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$\sim -R_2 + R_1 \rightarrow R_1 \begin{bmatrix} -3 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\sim \frac{-1}{3}R_1 \rightarrow R_1 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\text{So } E_5 E_4 E_3 E_2 E_1 A = I$$

$$\text{So } A = E_1^{-1} E_2^{-1} E_3^{-1} E_4^{-1} E_5^{-1}$$

Question 2. §1.6 #3 (5 marks) Solve the system by inverting the coefficient matrix.

$$\begin{aligned} x_1 + 3x_2 + x_3 &= 4 \\ 2x_1 + 2x_2 + x_3 &= -1 \\ 2x_1 + 3x_2 + x_3 &= 3 \end{aligned}$$

$$Ax = b$$

$$\text{where } A = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 2 & 1 \\ 2 & 3 & 1 \end{bmatrix}$$

$$b = \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix}$$

$$[A | I]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 2 & 2 & 1 & 0 & 1 & 0 \\ 2 & 3 & 1 & 0 & 0 & 1 \end{array} \right]$$

$$\sim \begin{array}{l} -2R_1 + R_2 \rightarrow R_2 \\ -2R_1 + R_3 \rightarrow R_3 \end{array} \left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & -4 & -1 & -2 & 1 & 0 \\ 0 & -3 & -1 & -2 & 0 & 1 \end{array} \right]$$

$$\sim \begin{array}{l} R_3 + R_1 \rightarrow R_1 \\ 4R_3 \rightarrow R_3 \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & -4 & -1 & -2 & 1 & 0 \\ 0 & -12 & -4 & -8 & 0 & 4 \end{array} \right]$$

$$\sim \begin{array}{l} -3R_2 + R_3 \rightarrow R_3 \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & -4 & -1 & -2 & 1 & 0 \\ 0 & 0 & -1 & -2 & -3 & 4 \end{array} \right]$$

$$\sim \begin{array}{l} -R_3 + R_2 \rightarrow R_2 \\ -R_3 \rightarrow R_3 \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & -4 & 0 & 0 & 4 & -4 \\ 0 & 0 & 1 & 2 & 3 & -4 \end{array} \right]$$

$$\sim \begin{array}{l} -\frac{1}{4}R_2 \rightarrow R_2 \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 2 & 3 & -4 \end{array} \right]$$

$$x = A^{-1}b$$

$$= \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 2 & 3 & -4 \end{bmatrix} \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ -7 \end{bmatrix}$$