Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S5: Fall 2014

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Quiz 6

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §1.7 #37 *(5 marks)* A square matrix A is said to be *skew-symmetric* if $A^T = -A$. Prove:

a. *(2 marks)* If A is an invertible skew-symmetric matrix, then A^{-1} is skew-symmetric.

b. *(3 marks)* If A and B are skew-symmetric matrices, then so are A^T, $A \pm B$, and kA for any scalar.

a) $$(A^{-1})^T = (A^T)^{-1} = (-A)^{-1} = \frac{1}{-1}A^{-1} = -A^{-1}$$

b) $$(A^T)^T = (-A)^T = -A^T$$

$$(A \pm B)^T = A^T \pm B^T = -A \pm (-B) = -(A \pm B)$$

$$(KA)^T = KA^T = K(-A) = -KA$$

Question 2. §2.1 #36 *(5 marks)* Show that

$$\det(A) = \frac{1}{2}\begin{vmatrix} \operatorname{tr}(A) & 1 \\ \operatorname{tr}(A^2) & \operatorname{tr}(A) \end{vmatrix}$$

for every 2×2 matrix A.

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $\det A = ad - bc$

and $\operatorname{tr} A = a + d$

$$A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix}$$

then $\operatorname{tr} A^2 = a^2 + bc + bc + d^2 = a^2 + 2bc + d^2$

$$\frac{1}{2}\begin{vmatrix} \operatorname{tr} A & 1 \\ \operatorname{tr}(A^2) & \operatorname{tr} A \end{vmatrix}$$

$$= \frac{1}{2}\left[(\operatorname{tr} A)^2 - \operatorname{tr}(A^2)\right]$$

$$= \frac{1}{2}\left[(a+d)^2 - (a^2 + 2bc + d^2)\right]$$

$$= \frac{1}{2}\left[\cancel{a^2} + 2ad + \cancel{d^2} - \cancel{a^2} - 2bc - \cancel{d^2}\right]$$

$$= ad - bc$$

$$= \det A$$