

Quiz 7

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §2.2 #29 (5 marks) Use row reduction to show that

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (b-a)(c-a)(c-b)$$

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} \xrightarrow{-aR_1 + R_2 \rightarrow R_2, -a^2R_1 + R_3 \rightarrow R_3} \begin{vmatrix} 1 & 1 & 1 \\ 0 & b-a & c-a \\ 0 & b^2-a^2 & c^2-a^2 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & b-a & c-a \\ 0 & (b-a)(b+a) & c^2-a^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 0 & b-a & c-a \\ 0 & 0 & c^2-a^2 - (b+a)(c-a) \end{vmatrix}$$

$$\begin{aligned} &= 1 \cdot (b-a) [c^2 - a^2 - (b+a)(c-a)] \\ &= (b-a) [(c-a)(c+a) - (b+a)(c-a)] \\ &= (b-a)(c-a) [c+a - (b+a)] \\ &= (b-a)(c-a)(c-b) \end{aligned}$$

Question 2. §2.3 #18 (5 marks) Find the values of k for which A is invertible

$$A = \begin{bmatrix} 1 & 2 & 0 \\ k & 1 & k \\ 0 & 2 & 1 \end{bmatrix} \begin{matrix} 1 & 2 \\ k & 1 \\ 0 & 2 \end{matrix}$$

$$|A| = 1 - 2k - 2k$$

A is invertible iff $|A| \neq 0$

$$1 - 4k \neq 0$$

$$1 \neq 4k$$

$$\frac{1}{4} \neq k$$