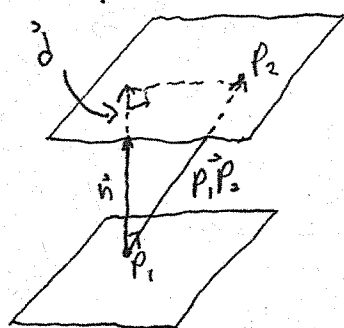


Quiz 8

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (5 marks) §3.3 #37 Use projections to find the distance between the given parallel planes.

$$\underbrace{2x - y - z = 5}_{P_1} \text{ and } \underbrace{-4x + 2y + 2z = 12}_{P_2}$$



$$\begin{aligned} \vec{P_1P_2} &= P_2 - P_1 \\ &= (-3, 0, 0) - (0, 0, -5) \\ &= (-3, 0, 5) \end{aligned}$$

$$\vec{n} = (2, -1, -1)$$

Let $x=y=0$ then $2(0) - 0 - z = 5 \Rightarrow z = -5$ is a point on P_1

Let $y=z=0$ then $-4x + 2(0) + 2(0) = 12 \Rightarrow x = -3$ is a point on P_2

$$\vec{d} = \text{proj}_{\vec{n}} \vec{P_1P_2}$$

$$= \frac{\vec{n} \cdot \vec{P_1P_2}}{\vec{n} \cdot \vec{n}} \vec{n}$$

$$= \frac{-3(2) + 0(-1) + 5(-1)}{2^2 + (-1)^2 + (-1)^2} (2, -1, -1)$$

$$= \frac{-11}{6} (2, -1, -1)$$

$$\text{distance} = \|\vec{d}\|$$

$$= \left\| \left(-\frac{11}{6} \right) (2, -1, -1) \right\|$$

$$= \frac{11}{6} \sqrt{2^2 + (-1)^2 + (-1)^2}$$

$$= \frac{11}{6} \sqrt{6}$$

Question 2.

a. (2 marks) §3.2 #7 Let $\vec{v} = (-2, 3, 0, 6)$. Find all scalars k such that $\|k\vec{v}\| = 5$

b. (3 marks) §3.2 #24c Find the radian measure of the angle θ (with $0 \leq \theta \leq \pi$) between $\vec{u} = (-1, 1, 0)$ and $\vec{v} = (0, -1, 1)$

$$a) \|k\vec{v}\| = 5$$

$$|k| \|\vec{v}\| = 5$$

$$|k| = \frac{5}{\|\vec{v}\|}$$

$$k = \pm \frac{5}{\|\vec{v}\|} = \pm \frac{5}{7}$$

$$\|\vec{v}\| = \sqrt{(-2)^2 + 3^2 + 0^2 + 6^2}$$

$$= \sqrt{4 + 9 + 36}$$

$$= \sqrt{49}$$

$$= 7$$

b)

$$\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos \theta$$

$$\cos \theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \|\vec{v}\|}$$

$$= \frac{(-1)(0) + 1(-1) + (0)(1)}{\sqrt{2} \sqrt{2}}$$

$$= -\frac{1}{2}$$

$$\theta = \arccos\left(-\frac{1}{2}\right)$$

$$= \frac{2\pi}{3}$$