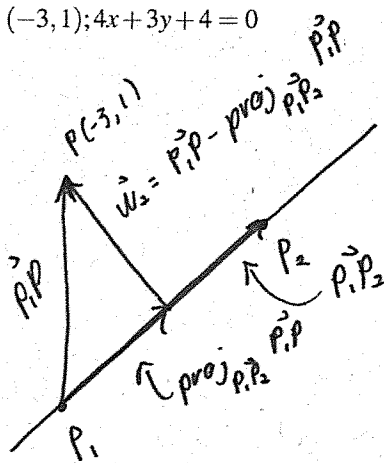


Quiz 9

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (5 marks) §3.3 #29 Using projections, find the distance between the point and the line.

$(-3, 1); 4x + 3y + 4 = 0$



Let $y = 0$
 $4x + 3(0) + 4 = 0$
 $x = -1$

$\therefore P_1(-1, 0)$

Let $x = -4$
 $4(-4) + 3y + 4 = 0$
 $3y = 12$
 $y = 4$

$\therefore P_2(-4, 4)$

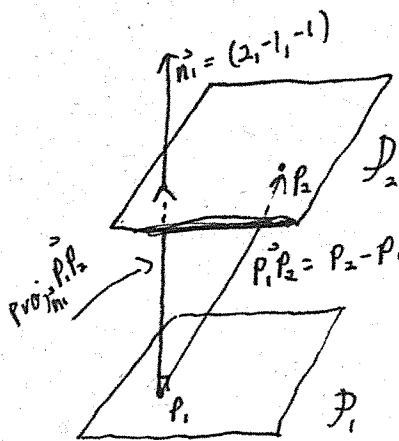
$\vec{P_1P_2} = P_2 - P_1 = (-4, 4) - (-1, 0) = (-3, 4)$
 $\vec{P_1P} = P - P_1 = (-3, 1) - (-1, 0) = (-2, 1)$

$\vec{w}_2 = \vec{P_1P} - \text{proj}_{\vec{P_1P_2}} \vec{P_1P}$
 $= (-2, 1) - \frac{(-3, 4) \cdot (-2, 1)}{(-3, 4) \cdot (-3, 4)} (-3, 4)$
 $= (-2, 1) - \frac{6 + 4}{9 + 16} (-3, 4)$
 $= (-2, 1) - \frac{10}{25} (-3, 4)$
 $= (-2, 1) - \frac{2}{5} (-3, 4) = \left(-\frac{4}{5}, \frac{-3}{5}\right)$

distance = $\|\vec{w}_2\|$
 $= \sqrt{\left(-\frac{4}{5}\right)^2 + \left(\frac{-3}{5}\right)^2} = 1$

Question 2. (5 marks) §3.3 #37 Using projections, find the distance between the given parallel planes.

$2x - y - z = 5$ and $-4x + 2y + 2z = 12$



Let $y = z = 0$
 $-4x + 2(0) + 2(0) = 12$
 $x = -3$
 $\therefore (-3, 0, 0)$

$\vec{P_1P_2} = P_2 - P_1 = (-3, 0, 0) - (0, 0, -5)$
 $= (-3, 0, 5)$

$\text{proj}_{\vec{n}_1} \vec{P_1P_2} = \frac{\vec{n}_1 \cdot \vec{P_1P_2}}{\vec{n}_1 \cdot \vec{n}_1} \vec{n}_1$
 $= \frac{(2, -1, -1) \cdot (-3, 0, 5)}{(2, -1, -1) \cdot (2, -1, -1)} (2, -1, -1)$
 $= \frac{-6 - 5}{4 + 1 + 1} (2, -1, -1)$
 $= \frac{-11}{6} (2, -1, -1)$

Let $x = y = 0$
 $2(0) - 0 - z = 5$
 $z = -5$
 $\therefore (0, 0, -5)$

distance = $\|\text{proj}_{\vec{n}_1} \vec{P_1P_2}\|$
 $= \left\| \frac{-11}{6} (2, -1, -1) \right\|$
 $= \frac{11}{6} \sqrt{2^2 + (-1)^2 + (-1)^2} = \frac{11}{6} \sqrt{6}$