Dawson College: **Linear Algebra**: 201-NYC-05-S05: Fall 2014

Name: ____________________
Student ID: ____________________

Test 2

This test is graded out of 40 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Given

$$A = \begin{bmatrix} 9 & 1 & 1 & 1 & 9 & 3 & 4 & 1 & 0 & 9 \\ 9 & 9 & 1 & 1 & 4 & 9 & 2 & 7 & 7 & 9 \\ 0 & 0 & 7 & 1 & 1 & 4 & 9 & 2 & 7 & 7 \\ 0 & 0 & 0 & 7 & 1 & 1 & 4 & 9 & 2 & 7 \\ 0 & 0 & 0 & 0 & 2 & 1 & 1 & 4 & 9 & 2 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 4 & 9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 1 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} -2 & 3 & 2 & 6 \\ 1 & 2 & 3 & -3 \\ -3 & 0 & 1 & 9 \\ 4 & 5 & 5 & -12 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} -2 & 3 & 2 & 3 \\ 1 & 0 & 3 & 0 \\ -3 & 0 & 1 & 2 \\ 4 & 5 & 0 & -1 \end{bmatrix}$$

a. *(3 marks)* Is A invertible, justify.

b. *(2 marks)* Is B invertible, justify.

c. *(2 marks)* Compute the determinant of C.

d. *(3 marks)* Compute the determinant of D.

Question 2. Consider

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 1 & -2 \\ 0 & 0 & 3 \end{bmatrix}.$$

a. *(4 marks)* Find A^{-1} using the adjoint of A.

b. *(4 marks)* Evaluate $\det(\det(A)\,\mathrm{adj}(A^{-1}))$, justify fully.

c. *(2 marks)* Using a. solve $Ax = b$ where $x = [x_1, x_2, x_3]^t$ and $b = [1, -2, 1]^t$.

Question 3. *(3 marks)* Solve *only* for x_3 using Cramer's rule.

$$\begin{array}{rcrcrcrcrcr}
3x_1 & - & x_2 & + & x_3 & - & 3x_4 & + & 4x_5 & = & 2 \\
 & & 3x_2 & & & & & - & 10x_5 & = & 3 \\
 & & & - & x_3 & - & 2x_4 & + & 3x_5 & = & -3 \\
 & & & & & - & x_4 & - & 3x_5 & = & 0 \\
 & & & & & & & & 4x_5 & = & 0
\end{array}$$

Question 4. Given $P(0,2,4)$ and $Q(2,5,7)$

a. *(2 marks)* Sketch the vector $\vec{PQ}$ with the initial point located at the origin.

b. *(3 marks)* Find the radian measure of the angle θ (with $0 \leq \theta \leq \pi$) between $\vec{u} = (2,-1,0)$ and $\vec{PQ}$

Question 5. *(2 marks)* Prove or Disprove: The linear combinations $a_1\vec{v_1} + a_2\vec{v_2}$ and $b_1\vec{v_1} + b_2\vec{v_2}$ can only be equal if $a_1 = b_1$ and $a_2 = b_2$.

Question 6. *(2 marks)* Let A and B be $n \times n$ matrices, $AB = -BA$, and n is odd, show that either A or B has no inverse.

Question 7. *(2 marks)* Show that $||\vec{u}+\vec{v}||^2 + ||\vec{u}-\vec{v}||^2 = 2(||\vec{u}||^2 + ||\vec{v}||^2)$. *Bonus (1 mark): what does that say about parallelograms?*

Question 8. *(2 marks)* Prove: If B and C are $n \times n$ matrices, $A = B^T C + C^T B$ is invertible then A^{-1} is symmetric.

Question 9. *(4 marks)* Write the given matrix as a product of elementary matrices

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 3 & 0 \\ 2 & 4 & 0 \end{bmatrix}$$

Bonus Question. *(4 marks)* Show that if the diagonals of a parallelogram are perpendicular, it is necessarily a rhombus.