Name: ____________________
Student ID: ____________________

Test 3

This test is graded out of 40 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Given

$A(1,2),\ B(-1,1),\ C(3,1)$

a. *(3 marks)* Find the area of the parallelogram generated by $\vec{AB}$ and $\vec{AC}$.

b. *(1 mark)* Find the area of the triangle with vertices A, B and C.

c. *(2 marks)* Find the length of the altitude of the triangle ABC from vertex C to side AB.

d. *(1 mark)* Find the shortest distance from C to the line that contains A and B.

Question 2. Given

$$\begin{array}{lrcll}
\mathscr{L}_1: & (x,y,z) & = & (2+5t, 1+t, -t) & t \in \mathbb{R} \\
\mathscr{L}_2: & (x,y,z) & = & (7+2t, 4, 10t) & t \in \mathbb{R} \\
\mathscr{L}_3: & (x,y,z) & = & (9-t, 2, 9-5t) & t \in \mathbb{R} \\
\mathscr{P}_1: & x-2y+3z-11 & = & 0 & \\
\mathscr{P}_2: & -5x-y+z+31 & = & 0 &
\end{array}$$

a. *(2 marks)* Are $\mathscr{P}_2$ and $\mathscr{L}_3$ parallel, perpendicular, or neither, justify? If they intersect find the point of intersection.

b. *(4 marks)* Are $\mathscr{L}_2$ and $\mathscr{P}_1$ parallel, perpendicular, or neither, justify? If they intersect find the point of intersection.

c. *(5 marks)* Are $\mathscr{L}_1$ and $\mathscr{L}_2$ parallel, perpendicular, or neither, justify? Find the shortest distance between the two lines using projections.

Question 4. Let $\mathscr{H} = \left\{ A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \,\middle|\, A \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$ with the usual addition and scalar multiplication.

a. *(2 marks)* Give an example of a non-zero matrix in $\mathscr{H}$? Justify.

b. *(2 marks)* Does $\mathscr{H}$ satisfy closure under vector addition? Justify.

c. *(2 marks)* Does $\mathscr{H}$ contain the zero vector of $\mathscr{M}_{2\times 2}$ (the vector space of 2×2 matrices)? Justify.

d. *(2 marks)* Does $\mathscr{H}$ satisfy closure under scalar multiplication? Justify.

e. *(2 marks)* Is $\mathscr{H}$ a vector subspace of $\mathscr{M}_{2\times 2}$ (the vector space of 2×2 matrices)? Justify.

Question 5. *(2 marks)* Prove or disprove: For all vector vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$ in 3-space, the vectors $(\vec{u} \times \vec{v}) \times \vec{w}$ and $\vec{u} \times (\vec{v} \times \vec{w})$ are the same.

Question 6. *(2 marks)* Prove: If $\vec{a}$ and $\vec{b}$ are orthogonal vectors, then for every nonzero vector $\vec{u}$, we have

$$\mathrm{proj}_{\vec{a}}(\mathrm{proj}_{\vec{b}}(\vec{u}) = \vec{0}$$

Question 7. *(2 marks)* Show that if $\vec{u}$, $\vec{v}$, and $\vec{w}$ are vectors in $\mathbb{R}^3$ no which are collinear, then $\vec{u} \times (\vec{v} \times \vec{w})$ lies in the plane determined by $\vec{v}$ and $\vec{w}$.

Question 8. *(2 marks)* Prove: If $\vec{u}$, $\vec{v}$, and $\vec{w}$ are vectors in a vector space V and $\vec{u}+\vec{w} = \vec{v}+\vec{w}$, then $\vec{u} = \vec{v}$. (*cancellation law for vector addition*). **Show EVERY step and justify EVERY step with axiom names when an axiom is used**

Question 9. *(2 marks)* Determine wether the following is a vector space:

$$\mathscr{P} = \left\{a_0 + a_1x + a_2x^2 \mid a_0 + a_1 + a_2 = 1\right\}$$

with the usual polynomial addition and scalar multiplication. i.e.

$$p_1(x) + p_2(x) = (a_0 + a_1x + a_2x^2) + (b_0 + b_1x + b_2x^2) = (a_0 + b_0) + (a_1 + b_1)x + (a_2 + b_2)x^2$$

and

$$kp_1(x) = k(a_0 + a_1x + a_2x^2) = ka_0 + ka_1x + ka_2x^2$$

Bonus Question. *(5 marks)* If U and W are subspaces of a vector space V, let

$$U \cup W = \left\{\vec{v} \,\middle|\, \vec{v} \in U \text{ or } \vec{v} \in W\right\}.$$

Show that $U \cup W$ is a subspace if and only if $U \subseteq W$ or $W \subseteq U$ (where $A \subseteq B$ means A is a subset of B or is B).