Linear Algebra
201-NYC-05 01
Test 1

Dawson College
Math Department
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1. Consider the points $A(1,0,2)$, $B(2,3,1)$, and $C(-2,1,3)$.

[4] **a)** Find the area of the triangle made by A, B, and C.
[4] **b)** Find the scalar equation of the plane containing A, B, and C.
[4] **c)** Find the point in the line through B and C which is closest to A.

[4] **2.** Find the value(s) of the constant k so that the vector $(3,-1,2)$ is orthogonal to the vector $(k,k^2,-1)$.

[4] **3.** Find the point of intersection of the line $\mathbf{x}=(1,-5,3)+t(3,2,-1)$ with the plane $2x+3y-7z=11$.

[4] **4.** Find the vector equation of the line that is parallel to the line of intersection of the planes $2x+y-4z=0$ and $-x+2y+3z=-1$, and that goes through the point $A(-2,5,0)$.

5. Consider the vectors $\mathbf{u}=(1,2,3,4)$ and $\mathbf{v}=(-3,1,2,-2)$ in $\mathbb{R}^4$.

[4] **a)** Find the angle between $\mathbf{u}$ and $\mathbf{v}$.
[4] **b)** Find a vector of norm 5 in the direction opposite to $\mathbf{v}$.

[6] **6.** Consider the point $B(3,2,-2,-1)$ and the hyperplane $2x_1+x_3-2x_4=0$ in $\mathbb{R}^4$. Find the distance between the hyperplane and B, and the point in the hyperplane which is closest to B.

[6] **7.** Consider the lines $\mathbf{p}=(-5,-10,6)+t(2,4,-3)$ and $\mathbf{q}=(-4,13,-3)+s(-3,5,-1)$. Find the distance between them, and the closest points in each to the other.

[4] **8.** If $\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w})=4$, what is $\mathbf{w}\cdot(2\mathbf{v}\times\mathbf{u})$?

[4] **9.** Let $\mathbf{u}$ and $\mathbf{v}$ be vectors in $\mathbb{R}^n$. Prove the following statements and interpret geometrically by a sketch.

a) $\|\mathbf{u}\|=\|\mathbf{v}\|$ if and only if $(\mathbf{u}+\mathbf{v})$ is orthogonal to $(\mathbf{u}-\mathbf{v})$
b) $\|\mathbf{u}\|=\|\mathbf{v}\|$ if and only if $(\mathbf{u}+\mathbf{v})$ bisects the angle between $\mathbf{u}$ and $\mathbf{v}$.

[4] **10.** Prove that the line segments joining the midpoints of opposite sides of a quadrilateral bisect each other. (Hint: you can use anything already proved in class.)

[*Total Points = 56*]