Linear Algebra
201-NYC-05 01
Test 3

Dawson College
Math Department
May 5, 2014

[*7*] **1.** Find $\det\begin{pmatrix} 1 & 3 & 1 & 5 \\ 1 & 3 & -3 & -3 \\ 0 & 3 & 1 & 0 \\ 1 & 6 & 2 & 11 \end{pmatrix}$ any way you like.

[*7*] **2.** Find the inverse of $\begin{pmatrix} 2 & 3 & -5 \\ 3 & -1 & 2 \\ 5 & 4 & -6 \end{pmatrix}$ using the adjoint formula.

[*6*] **3.** Use Cramer's Rule to solve **only for** y in the system $\begin{cases} 2x + 3y + z = 1 \\ x + y - z = -1 \\ -2x + 2z = 1 \end{cases}$.

[*6*] **4.** Suppose A and B are 4×4 matrices, with $\det(A) = 3$. Find $\det(B)$ given that

$$\det(2A^T B^{-1}) = \det(\operatorname{adj}(A)B)$$

[*6*] **5.** Let A be an $n \times n$ matrix.

a) If $A^T = -A$ and n is odd, show that A is not invertible.
b) If $A^T = -A$ and n is even, what can you say about the invertibility of A?
c) If $A^2 + I = 0$, show that n is even.

[*4*] **6.** Let $V = \mathbb{R}^2$, with the usual scalar multiplication, but define addition by $(x_1, x_2) + (y_1, y_2) = (x_2 + y_2, x_1 + y_1)$. Show that V under these operations is not a vector space. Clearly state which axiom fails.

[*8*] **7.** For an $m \times n$ matrix A, let $\ker(A) = \{\mathbf{x} \in \mathbb{R}^n \mid A\mathbf{x} = \mathbf{0}\}$. Show that

a) $\ker(A)$ is a subspace of $\mathbb{R}^n$.
b) $\ker(A) = \{\mathbf{0}\}$ if and only if the columns of A are linearly independent.
c) if $m < n$, then $\ker(A) \neq \{\mathbf{0}\}$.
d) if $A = BC$, where B is $m \times m$ and C is $m \times n$, and if B is invertible, then $\ker(A) = \ker(C)$.

[*4*] **8.** For which value(s) of t will the set $\{(1, 1, 0), (1, 3, -1), (5, 3, t)\}$ be linearly independent?

[*4*] **9.** Let $\alpha = \{(1, 1, 1), (1, 0, -1), (1, 0, 1)\}$ and $\beta = \{(1, 2, 1), (2, 3, 4), (3, 4, 3)\}$ be two bases for $\mathbb{R}^3$. If $(\mathbf{x})_\alpha = (5, -1, -1)$, what is $(\mathbf{x})_\beta$?

[*4*] **10.** Find a basis for, and the dimension of, the kernel of $\begin{pmatrix} 1 & 0 & 2 & 0 & 4 \\ 0 & 1 & 3 & 0 & 5 \\ 0 & 0 & 0 & 1 & 6 \end{pmatrix}$.

[*Total Points = 56*]

[*3*] **B1.** Find a basis for, and the dimension of, the space of matrices that commute with $A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$. (i.e. the space of all matrices X such that $AX = XA$)

[*4*] **B2.** Prove the commutativity axiom, assuming the other 9 vector space axioms.