Dawson College: Introduction to Applied Mathematics: 201-912-DW: Fall 2015

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Test 1

This test is graded out of 50 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page. This test consists of 9 questions and one bonus question. The maximum possible grade is 50/50. Please ensure that you have a complete test. This test must be returned intact.

Question 1. *(5 marks)* Simplify the following expressing your final answer with positive exponents only:

$$\frac{(x^3y^{-2})^{-2}z^5}{x^{-8}(yz)^2}\cdot\left(\frac{x^3}{z^{-2}}\right)^{-4}$$

$$= \frac{(x^3)^{-2}(y^{-2})^{-2}z^5}{x^{-8}y^2z^2}\cdot\frac{(x^3)^{-4}}{(z^{-2})^{-4}}$$

$$= \frac{x^{-6}y^4z^5}{x^{-8}y^2z^2}\cdot\frac{x^{-12}}{z^8}$$

$$= \frac{x^8y^2z^3}{x^6x^{12}z^8}$$

$$= \frac{x^2y^2z^3}{x^{20}z^{85}}$$

$$= \frac{y^2}{x^{10}z^5}$$

Question 2. Convert the following, show all your work and *use the correct number of significant figures*

a. *(2 marks)* 134500 ft·lb bending moment to MN·m .

b. *(2 marks)* 123.1kPa to psi (pound per square inch).

a) $134500\ \text{ft}\cdot\text{lb}\cdot\frac{4.448\text{N}}{1\ \text{lb}}\cdot\frac{12\ \text{in}}{1\text{ft}}\cdot\frac{2.54\text{cm}}{1\ \text{in}}\cdot\frac{10^{-2}}{1\text{c}}\cdot\frac{1\text{M}}{10^6} = 0.1823\ \text{MN}\cdot\text{m}$

b) $123.1\ \text{kPa}$

$= 123.1\ \text{KN}\cdot\text{m}^{-2}$

$= 123.1\ \frac{\text{KN}}{\text{m}^2}\cdot\frac{10^3}{1\text{K}}\cdot\frac{1\ \text{lb}}{4.448\text{N}}\cdot\frac{10^{-2}}{1\text{c}}\cdot\frac{10^{-2}}{1\text{c}}\cdot\frac{2.54\ \text{cm}}{1\text{in}}\cdot\frac{2.54\ \text{cm}}{1\ \text{in}}$

$= 17.86\ \frac{\text{lb}}{\text{in}^2}$

$= 17.86\ \text{psi}$

Question 3. Let $f(x) = \dfrac{1}{2x+5}$

a. Evaluate and simplify the following:

i. *(1 mark)* $f(\frac{1}{2})$

ii. *(1 mark)* $f(x-2)$

iii. *(1 mark)* $f(a+h)$

iv. *(1 mark)* $f(2)+x$

b. *(1 mark)* Find the domain of $f(x)$

ai) $f(\frac{1}{2}) = \dfrac{1}{2(\frac{1}{2})+5} = \dfrac{1}{1+5} = \dfrac{1}{6}$

ii) $f(x-2) = \dfrac{1}{2(x-2)+5} = \dfrac{1}{2x-4+5} = \dfrac{1}{2x+1}$

iii) $f(a+h) = \dfrac{1}{2(a+h)+5} = \dfrac{1}{2a+2h+5}$

iv) $f(2)+x = \dfrac{1}{2(2)+5} + x = \dfrac{1}{9} + x = \dfrac{1}{9} + \dfrac{9x}{9} = \dfrac{1+9x}{9}$

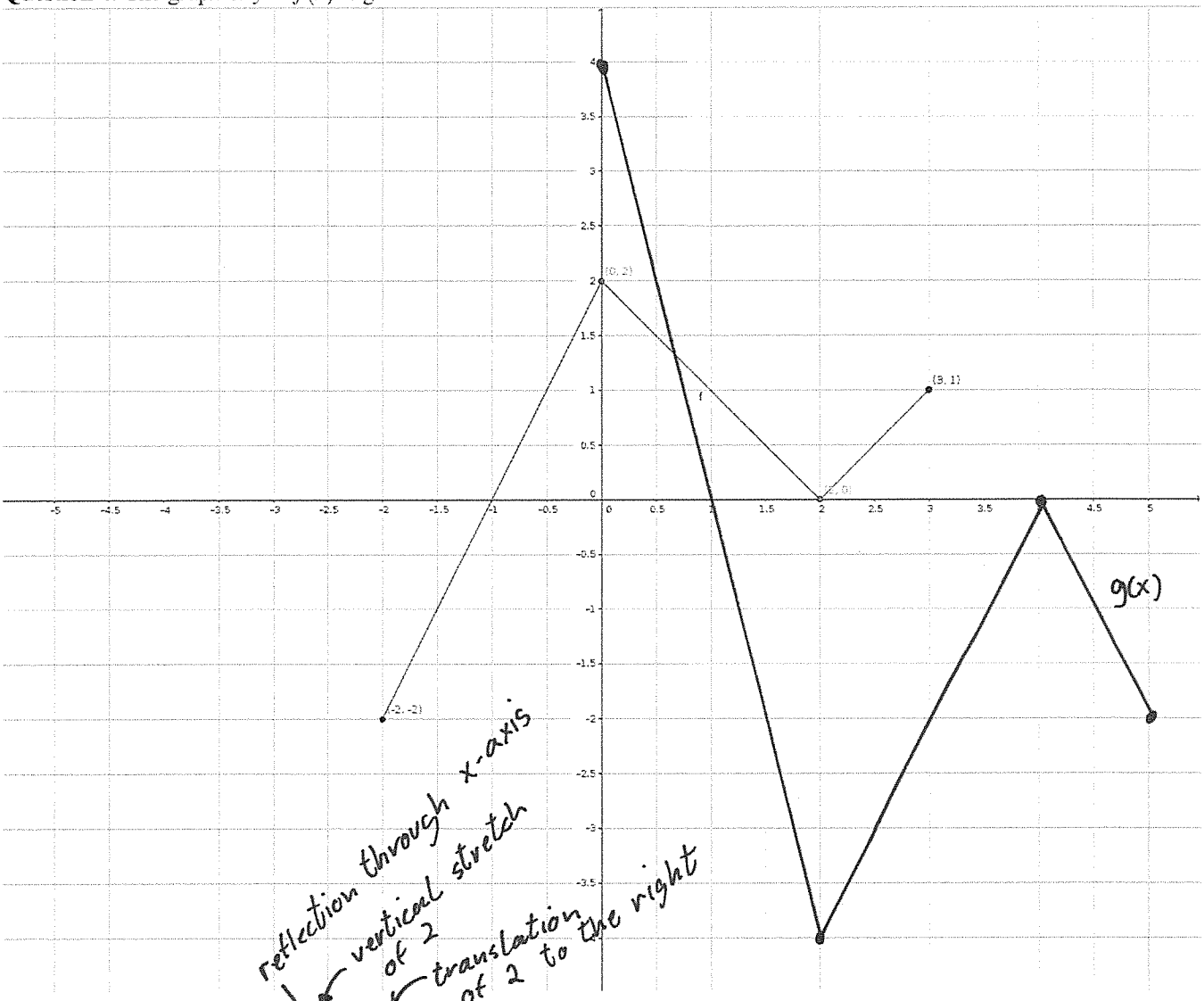
b) $2x+5 \neq 0$

$2x \neq -5$

$x \neq -5/2$

$\therefore$ the domain is $\mathbb{R} \setminus \{-5/2\} \equiv (-\infty, -5/2) \cup (-5/2, \infty)$

Question 4. The graph of $y = f(x)$ is given below.



a. *(3 marks)* Graph $g(x) = -2f(x-2)$ on the given set of axes.

b. *(1 mark)* State the domain and range of g(x).

domain: $[0,5]$

range : $[-4,4]$

Question 5. Let $f(x) = \frac{1}{x-5}$, $g(x) = x^2 + 1$.

a. *(2 marks)* Simplify the expression $(f \circ g)(x)$.

b. *(1 mark)* Evaluate $(f \circ g)(0)$, if possible.

c. *(1 mark)* Evaluate $(f \circ g)(2)$, if possible.

d. *(2 marks)* State the domain of $f \circ g$.

a) $(f \circ g)(x) = f(g(x))$

$= f(x^2+1)$

$= \frac{1}{x^2+1-5}$

$= \frac{1}{x^2-4}$

b) $(f \circ g)(0) = \frac{1}{0^2-4} = -\frac{1}{4}$

c) Not defined since $x=2$ makes the denominator equal to zero.

d) $(f \circ g)(x) = \frac{1}{x^2-4}$

$x^2 - 4 \neq 0$

$(x-2)(x+2) \neq 0$

$x \neq 2 \quad x \neq -2$

∴ the domain is $\mathbb{R} \setminus \{-2, 2\}$ and $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

Question 6. For each of the following functions, find the inverse $f^{-1}(x)$ and state the domain of $f^{-1}(x)$

a. *(3 marks)* $f(x) = \frac{1}{2}x^3 + 5$

b. *(4 marks)* $f(x) = 3 - \frac{1}{2}\sqrt{x-1}$

a) $y = \frac{1}{2}x^3 + 5$

$x = \frac{1}{2}y^3 + 5$

$x - 5 = \frac{1}{2}y^3$

$y^3 = 2(x-5)$

$y = \sqrt[3]{2(x-5)}$

$\therefore f^{-1}(x) = \sqrt[3]{2(x-5)}$

$\therefore$ the domain of $f^{-1}(x)$ is $\mathbb{R}$

b) $y = 3 - \frac{1}{2}\sqrt{x-1}$

$x = 3 - \frac{1}{2}\sqrt{y-1}$

$\frac{1}{2}\sqrt{y-1} = 3 - x$

$\sqrt{y-1} = 2(3-x)$

$y - 1 = (2(3-x))^2$

$y = (2(3-x))^2 + 1$

$\therefore f^{-1}(x) = (2(3-x))^2 + 1$

$\therefore$ the range of $f(x)$ is $(-\infty, 3]$

$\therefore$ the domain of $f^{-1}(x)$ is $(-\infty, 3]$

Question 7. A linear function $f(x)$ has a slope of -2 and passes through the point $(2, 1)$.

a. *(2 marks)* Determine the linear function $f(x)$.

b. *(2 marks)* Sketch the graph of $f(x)$.

c. *(2 marks)* Clearly state and label the x and y intercept on the graph.

a) $f(x) = mx + b$

$f(x) = -2x + b$

$f(2) = 1$

$1 = -2(2) + b$

$1 + 4 = b$

$5 = b$

$\therefore f(x) = -2x + 5.$

c) x-int: $0 = f(x)$

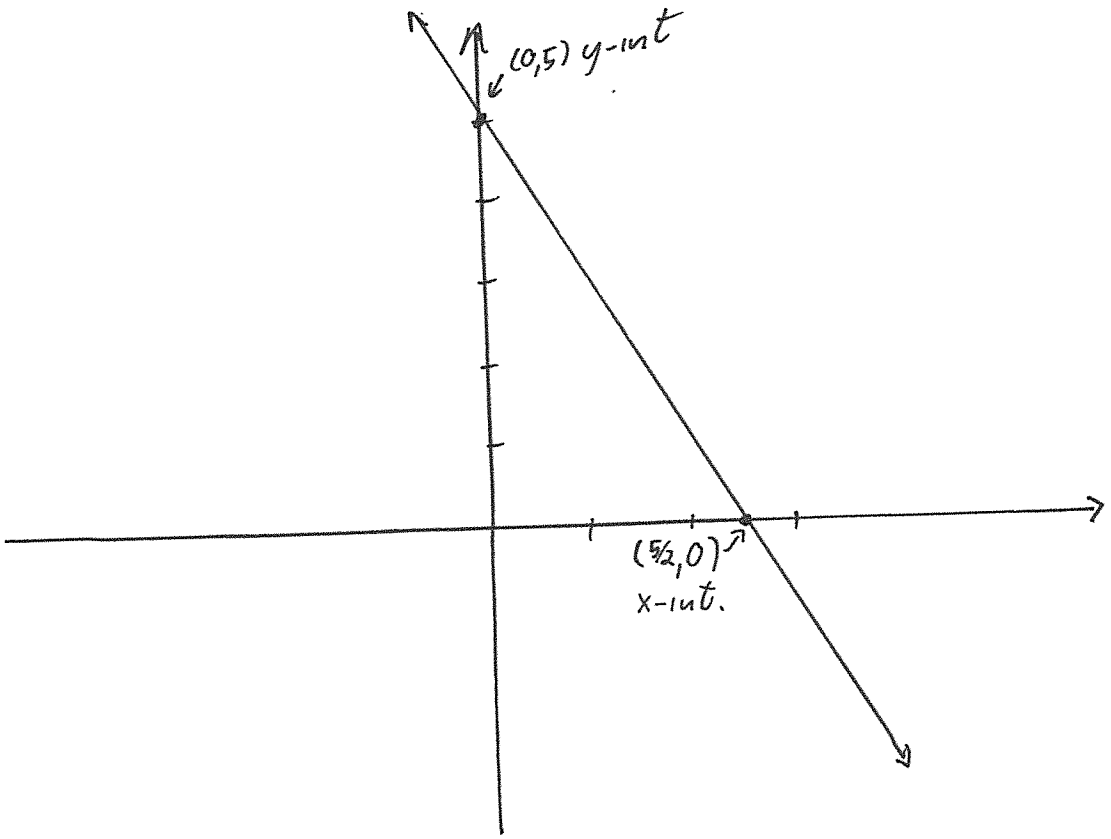
$0 = -2x + 5$

$x = 5/2$

$\therefore$ x-int. is $(5/2, 0)$

y-int: $(0, 5)$

b)



Question 8. Consider the quadratic function $g(x) = -2x^2 + 10x - 8$.

a. *(2 marks)* Determine the x and y intercept of $g(x)$.

b. *(2 marks)* Determine the vertex of $g(x)$.

c. *(2 marks)* Sketch the graph of $g(x)$ and label the vertex, x and y intercept on the graph.

d. *(1 mark)* State the domain and range of $g(x)$.

a) y-int.: $(0, g(0)) = (0, -8)$

x-int.: $0 = g(x)$

$0 = -2x^2 + 10x - 8$

$0 = x^2 - 5x + 4$

$0 = (x-1)(x-4)$

$x = 1$ $\quad$ $x = 4$

b) $g(x) = -2x^2 + 10x - 8$

$= -2[x^2 - 5x + 4]$

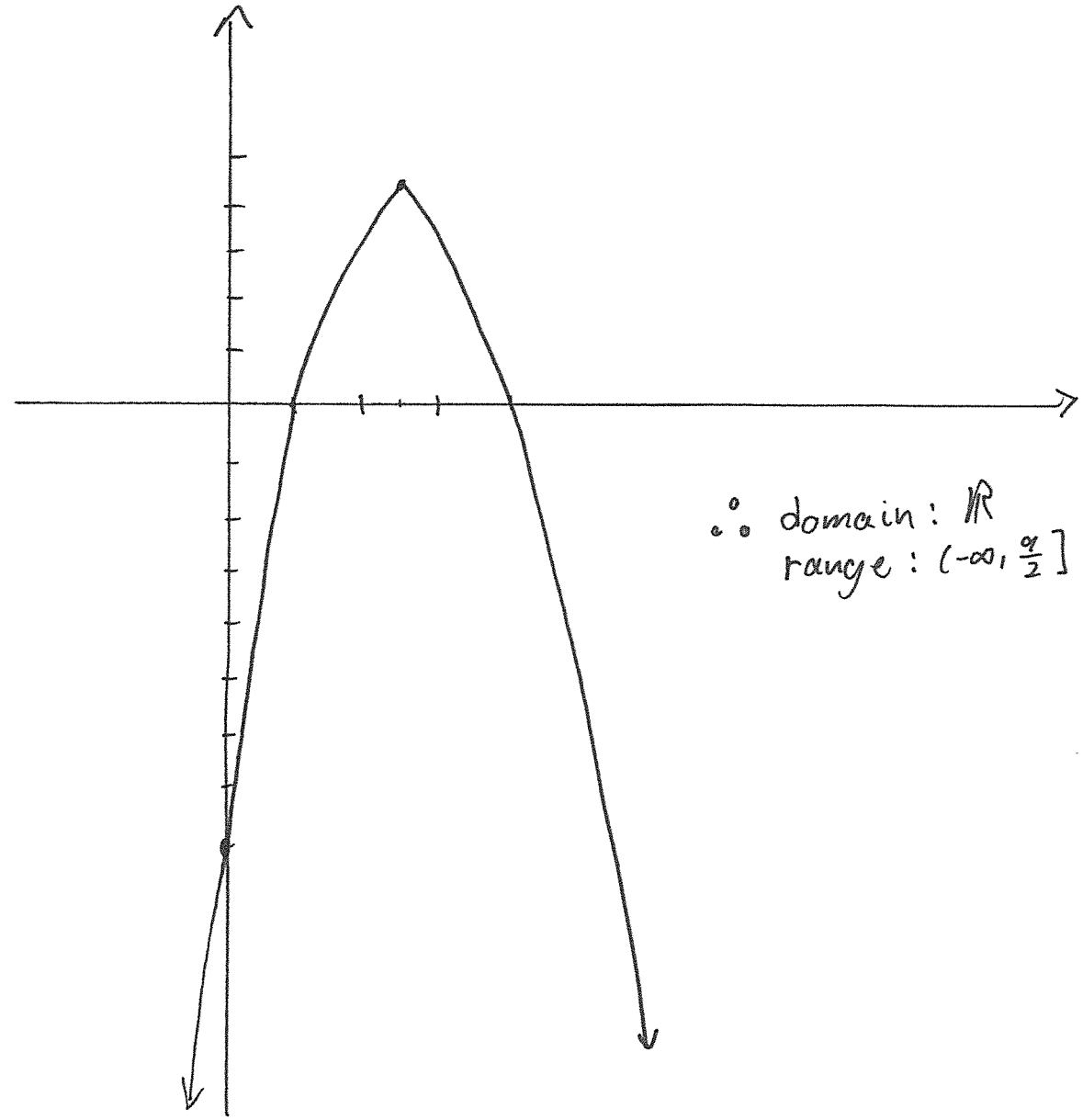
$= -2\left[x^2 - 5x + \frac{25}{4} - \frac{25}{4} + 4\right]$

$= -2\left[\left(x - \frac{5}{2}\right)^2 - \frac{9}{4}\right]$

$= -2\left(x - \frac{5}{2}\right)^2 + \frac{9}{2}$

$\therefore$ the vertex is $\left(\frac{5}{2}, \frac{9}{2}\right)$

c)



$\therefore$ domain: $\mathbb{R}$

range: $(-\infty, \frac{9}{2}]$

Question 9. In room 4H.6 of Dawson College, the percentage humidity in the air H is measured the afternoon hours of a summer day.*(fictional data)*

t	0.0	1.0	2.0	3.0	4.0	5.0
H	60	63	67	68	72	71

a. *(4 marks)* Find the least-squares line for H in percentage as a function of the time t in hours from noon.

b. *(2 marks)* At what time of the day can we predict the humidity in room 4H.6 to be 66%.

a)

$t(x)$	$H(y)$	xy	x^2
0.0	60	0	0
1.0	63	63	1
2.0	67	134	4
3.0	68	204	9
4.0	72	288	16
5.0	71	355	25
15.0	401	1044	55

$$m = \frac{n\Sigma xy - (\Sigma x)(\Sigma y)}{n\Sigma x^2 - (\Sigma x)^2} = \frac{6(1044) - 15(401)}{6(55) - (15)^2}$$

$$= \frac{249}{105} = \frac{83}{35}$$

$$b = \frac{(\Sigma x^2)(\Sigma y) - (\Sigma xy)(\Sigma x)}{n\Sigma x^2 - (\Sigma x)^2}$$

$$= \frac{55(401) - (1044)(15.0)}{6(55) - (15)^2} = \frac{6395}{105} = \frac{1279}{21}$$

$\therefore H = \frac{249}{105}t + \frac{6395}{105}$

b)

$$66 = \frac{249}{105}t + \frac{6395}{105}$$

$$249t = 66(105) - 6395$$

$$t = \frac{535}{249}$$

$\doteq 2.15$ hours

$\therefore$ at 14h09.

Bonus Question. *(3 marks)*
A function f is called <u>even</u> if $f(-x) = f(x)$ for all x in the domain of f.
A function f is called <u>odd</u> if $f(-x) = -f(x)$ for all x in the domain of f.
Find a function which is both even and odd.

$f(x) = 0$