Dawson College: **Introduction to Applied Mathematics**: 201-912-DW: Fall 2015

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Test 2

This test is graded out of 50 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page. This test consists of 9 questions and one bonus question. The maximum possible grade is 50/50. Please ensure that you have a complete test. This test must be returned intact.

Question 1. Newton's Law of Cooling (Warming)[1]: The temperature T of an object at time t is given by the formula

$$T(t) = T_a + (T_0 - T_a)e^{-kt}$$

where $T(0) = T_0$ is the initial temperature of the object, T_a is the ambient temperature and $k > 0$ is the constant of proportionality.

A 50°C object is cooked in a 350°C oven. After 3 hours the temperature of the object is 90°C.

a. *(2 marks)* Assuming the temperature of the object follows Newton's Law of Warming, find a formula for the temperature of the object T as a function of its time in the oven, t, in hours.

b. *(2 marks)* The object is done cooking when the internal temperature reaches 225°C. After how many hours will the object be cooked?

a)
$$90 = 350 + (50-350)e^{-K3}$$
$$-260 = -300e^{-K3}$$
$$\frac{13}{15} = e^{-K3}$$
$$\ln\left(\frac{13}{15}\right) = \ln e^{-3K}$$
$$\ln\left(\frac{13}{15}\right) = -3K$$
$$K = \frac{\ln\left(\frac{13}{15}\right)}{-3}$$
$$\therefore T(t) = 350 - 300e^{\frac{\ln\left(\frac{13}{15}\right)}{3}t}$$

b)
$$225 = 350 - 300e^{\frac{\ln\left(\frac{13}{15}\right)}{3}t}$$
$$-125 = -300e^{\frac{\ln\left(\frac{13}{15}\right)}{3}t}$$
$$\frac{5}{12} = e^{\frac{\ln\left(\frac{13}{15}\right)}{3}t}$$
$$\ln\left(\frac{5}{12}\right) = \ln e^{\frac{\ln\left(\frac{13}{15}\right)}{3}t}$$
$$\ln\left(\frac{5}{12}\right) = \frac{\ln\left(\frac{13}{15}\right)}{3}t$$
$$t = \frac{3\ln\left(\frac{5}{12}\right)}{\ln\left(\frac{13}{15}\right)}$$
$$\doteq 18 \text{ hours}$$

Question 2.[2] *(4 marks)* Solve for x.

$$\log_3(x-4) + \log_3(x+4) = 2$$

$$\log_3(x-4)(x+4) = 2$$
$$3^{\log_3(x-4)(x+4)} = 3^2$$
$$(x-4)(x+4) = 9$$
$$x^2 - 16 = 9$$
$$x^2 = 25$$
$$x = \pm 5$$

$x = -5$ is not a solution since $\log_3(-5-4)$ is not defined.

$$\therefore x = 5$$

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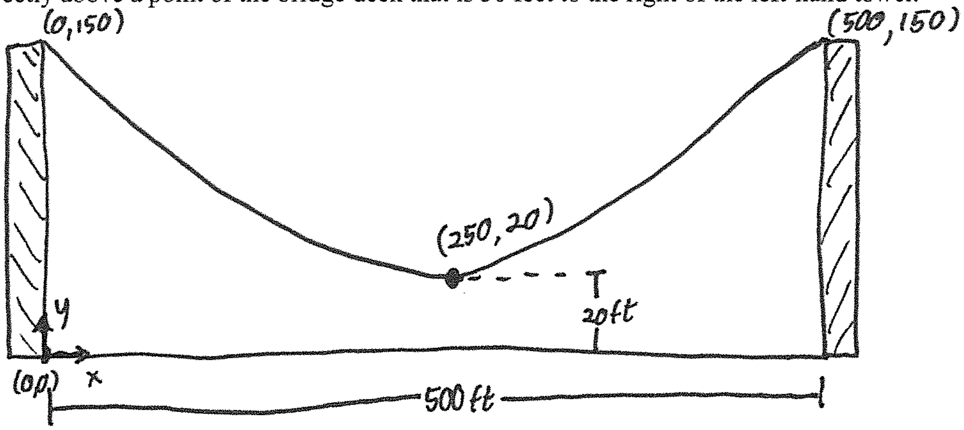
[1] from Precalculus, version 3, Carl Stitz and Jeff Zeage, 2011

[2] from Precalculus, version 3, Carl Stitz and Jeff Zeage, 2011

Question 3.[3] *(5 marks)* The two towers of a suspension bridge are 500 feet apart. The parabolic cable attached to the tops of the towers is 20 feet above the point on the bridge deck that is midway between the towers. If the towers are 150 feet tall, Find the height of the cable directly above a point of the bridge deck that is 50 feet to the right of the left-hand tower.



vertex at $(250, 20)$

$$\therefore \; f(x) = a(x-h)^2 + k$$
$$= a(x-250)^2 + 20$$

And $(0,150)$ is a point on the parabola so

$$150 = a(0-250)^2 + 20$$
$$130 = a(250)^2$$
$$a = \frac{130}{(250)^2} = \frac{13}{6250}$$

$$\therefore \; f(x) = \frac{13}{6250}(x-250)^2 + 20$$

The height of the cable at 50ft is $f(50) = \frac{13}{6250}(50-250)^2 + 20$

$$= \frac{516}{5}$$
$$= 103.2 \text{ ft.}$$

[3]modified from Precalculus, version 3, Carl Stitz and Jeff Zeage, 2011

Question 4. *(6 marks)* How much of each of the following mixtures must be used in order to make a 20m^3 mixture which has 15% sand, 35% aggregate and 50% cement.

Mix A: 10% sand, 30% aggregate and 60% cement
Mix B: 20% sand, 10% aggregate and 70% cement
Mix C: 10% sand, 80% aggregate and 10% cement

Total : $A + B + C = 20$
Sand : $10\%A + 20\%B + 10\%C = 15\%(20)$
Aggregate : $30\%A + 10\%B + 80\%C = 35\%(20)$
Cement : $60\%A + 70\%B + 10\%C = 50\%(20)$

$$\begin{cases} E_1 & A + B + C = 20 \\ E_2 & 0.1A + 0.2B + 0.1C = 3 \\ E_3 & 0.3A + 0.1B + 0.8C = 7 \\ E_4 & 0.6A + 0.7B + 0.1C = 10 \end{cases}$$

$$\sim \begin{array}{l} \\ -0.1E_1 + E_2 \to E_2 \\ -0.3E_1 + E_3 \to E_3 \\ -0.6E_1 + E_4 \to E_4 \end{array} \begin{cases} E_1 & A + B + C = 20 \\ E_2 & 0.1B = 1 \\ E_3 & -0.2B + 0.5C = 1 \\ E_4 & 0.1B - 0.5C = -2 \end{cases}$$

$$\sim \begin{array}{l} \\ \\ -2E_2 + E_3 \to E_3 \\ -E_2 + E_4 \to E_4 \end{array} \begin{cases} E_1 & A + B + C = 20 \\ E_2 & 0.1B = 1 \\ E_3 & 0.5C = 3 \\ E_4 & -0.5C = -3 \end{cases}$$

$$\sim \begin{array}{l} \\ \\ \\ E_3 + E_4 \to E_4 \end{array} \begin{cases} E_1 & A + B + C = 20 \\ E_2 & 0.1B = 1 \\ E_3 & 0.5C = 3 \\ E_4 & 0 = 0 \end{cases}$$

From E_3: $0.5C = 3$, $C = 6$

From E_2: $0.1B = 1$, $B = 10$

From E_1: $A + B + C = 20$, $A + 10 + 6 = 20$, $A = 4$.

Question 5. *(4 marks)* Solve the following linear system using Cramer's rule.

$$\begin{aligned} x_1 - 2x_2 &= 5 \\ 3x_1 - 4x_2 &= -6 \end{aligned}$$

$|A| = \begin{vmatrix} 1 & -2 \\ 3 & -4 \end{vmatrix} = 1(-4) - (-2)(3) = 2$

$|A_1| = \begin{vmatrix} 5 & -2 \\ -6 & -4 \end{vmatrix} = 5(-4) - (-2)(-6) = -22$

$|A_2| = \begin{vmatrix} 1 & 5 \\ 3 & -6 \end{vmatrix} = 1(-6) - 5(3) = -21$

$x_1 = \dfrac{|A_1|}{|A|} = \dfrac{-22}{2} = -11$

$x_2 = \dfrac{|A_2|}{|A|} = \dfrac{-21}{2}$

Question 6. *(4 marks)* Graph the function $f(x) = 4 - 2^{-x}$

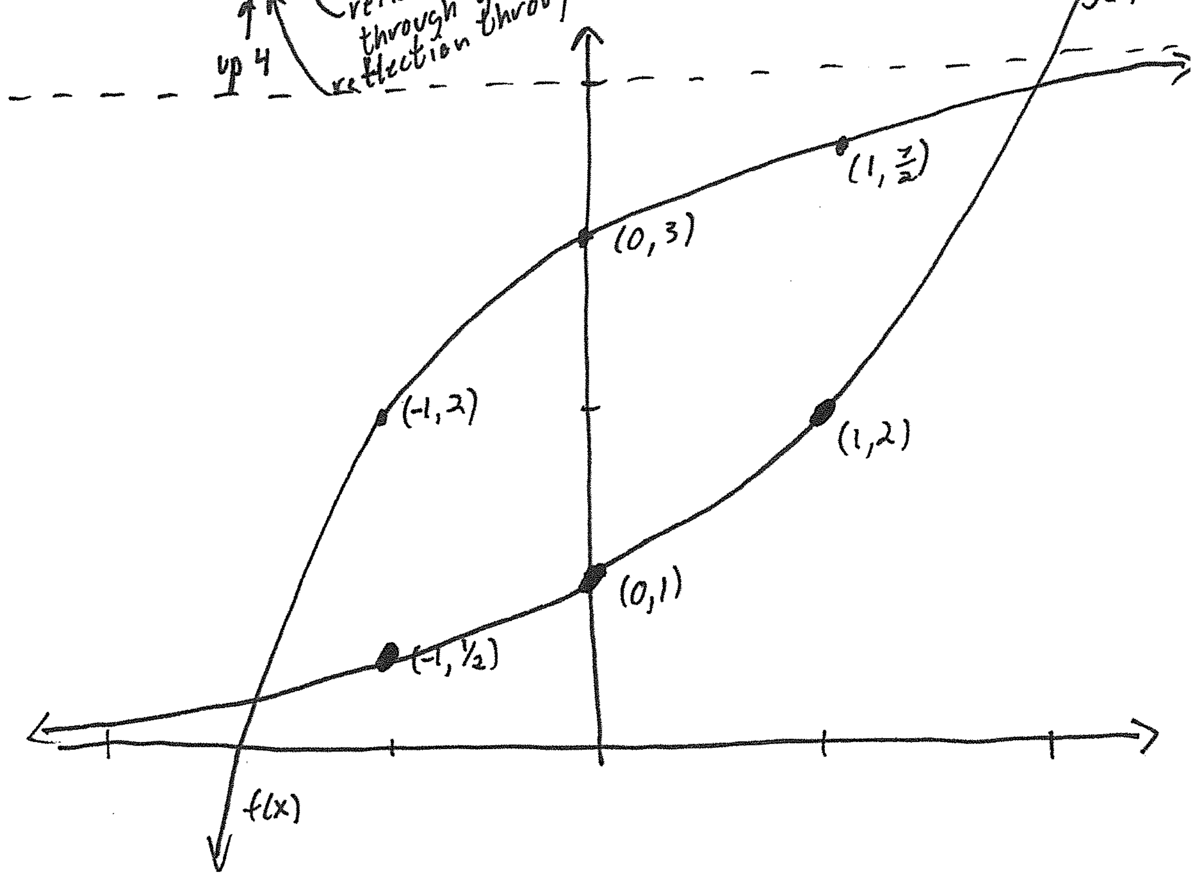
up 4

reflection through y-axis

reflection throug x-axis

Let $g(x) = 2^x$

x	$g(x)$
-1	$\frac{1}{2}$
0	1
1	2



Question 7.

a. *(4 marks)* Find the center and the radius of the circle. *hint: complete the square for the variable y*

$$x^2 + y^2 - 8y + 12 = 0$$

b. *(4 marks)* Find the standard equation of the ellipse $4x^2 + 9y^2 - 36 = 0$. Identify the center and vertices of the ellipse.

c. *(4 marks)* Solve the non-linear system

$$\begin{aligned} x^2 + y^2 - 8y + 12 &= 0 \\ 4x^2 + 9y^2 - 36 &= 0 \end{aligned}$$

d. *(3 marks)* Sketch both curves from c. on the same graph.

a) $x^2 + y^2 - 8y + 12 = 0$

$x^2 + [(y^2 - 8y + 16) - 16] + 12 = 0$

$x^2 + [(y-4)^2 - 16] + 12 = 0$

$x^2 + (y-4)^2 = 4$

$\therefore$ center $(0,4)$

radius 2

b) $4x^2 + 9y^2 = 36$

$\frac{4x^2}{36} + \frac{9y^2}{36} = 1$

$\frac{x^2}{9} + \frac{y^2}{4} = 1$

$\therefore$ center is $(0,0)$

Let $x=0$ $\quad \frac{y^2}{4} = 1$

$y^2 = 4$

$y = \pm 2$

Let $y=0$ $\quad \frac{x^2}{9} = 1$

$x^2 = 9$

$x = \pm 3$

$\therefore$ vertices are $(-3,0)$ and $(3,0)$

$\therefore$ (0, are $(0,-2)$ and $(0,2)$

sub $y=2$ in E_1

$x^2 + 2^2 - 8(2) + 12 = 0$

$x^2 = 0$

$x = 0$

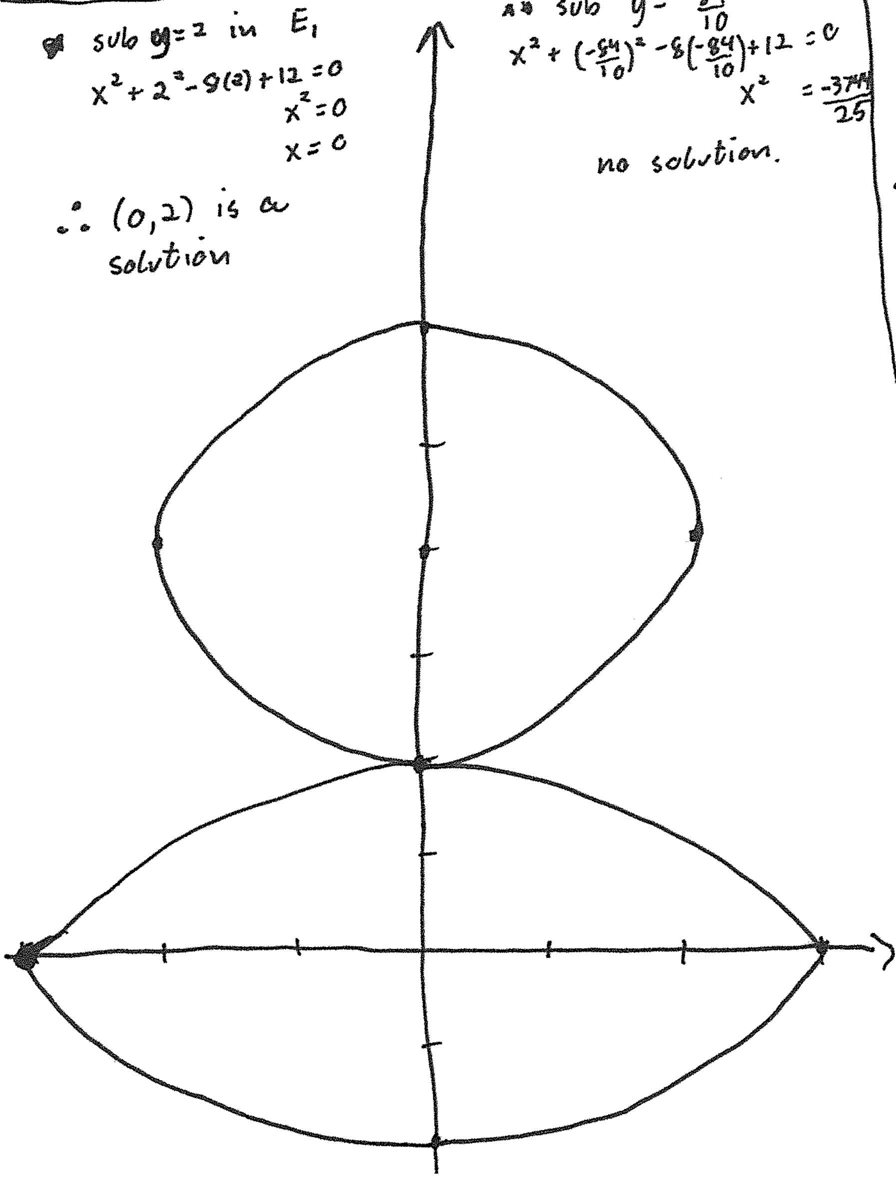
$\therefore (0,2)$ is a solution

sub $y = \frac{-84}{10}$

$x^2 + \left(\frac{-84}{10}\right)^2 - 8\left(\frac{-84}{10}\right) + 12 = 0$

$x^2 = \frac{-3744}{25}$

no solution.



c) E_1 $x^2 + y^2 - 8y + 12 = 0$

E_2 $4x^2 + 9y^2 - 36 = 0$

$-4E_1 + E_2$:

$5y^2 + 32y - 84 = 0$

$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-32 \pm \sqrt{(32)^2 - 4(5)(-84)}}{2(5)}$

$= \frac{-32 \pm 52}{10}$

$= \frac{20}{10}$ or $\frac{-84}{10}$

$= 2$ or $\frac{-84}{10}$

Goto and

Question 8. Expand the given logarithms and simplify as much as possible:

a. *(2 marks)*

$$\log_3\left(\frac{81\sqrt{x}\,y^5}{z}\right) = \log_3 81\sqrt{x}y^5 - \log_3 z = \log_3 81 + \log_3 \sqrt{x} + \log_3 y^5 - \log_3 z$$
$$= 4 + \frac{1}{2}\log_3 x + 5\log_3 y - \log_3 z$$

b. *(2 marks)*

$$\ln\left(x^2(x+3)\right)$$
$$= \ln x^2 + \ln(x+3)$$
$$= 2\ln x + \ln(x+3)$$

Question 9. Use the properties of logarithms to write each expression as a single logarithm:

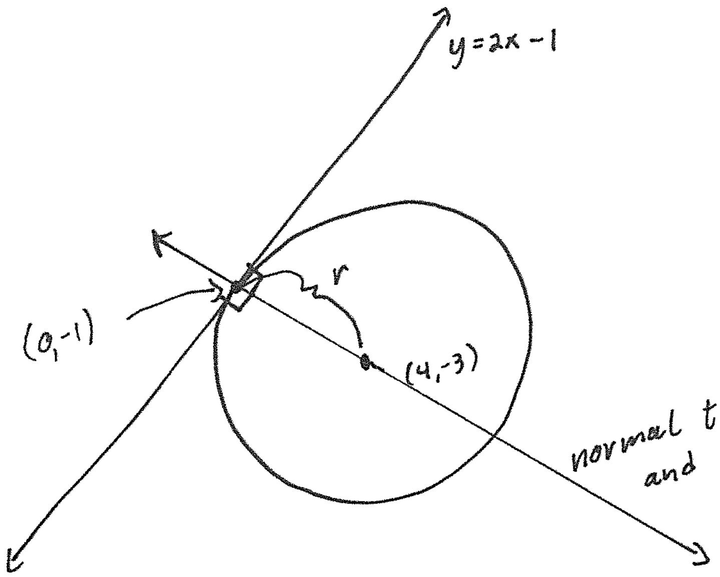
a. *(2 marks)*

$$3\ln x + \frac{1}{3}\ln y - 2\ln z = \ln x^3 + \ln\sqrt[3]{y} - \ln z^2 = \ln\frac{x^3\sqrt[3]{y}}{z^2}$$

b. *(2 marks)*

$$\log_5 x - \log_{25} x$$
$$= \log_5 x - \frac{\log_5 x}{\log_5 25}$$
$$= \log_5 x - \frac{\log_5 x}{2}$$
$$= \log_5 x - \frac{1}{2}\log_5 x$$
$$= \log_5 x - \log_5 \sqrt{x}$$
$$= \log_5\left(\frac{x}{\sqrt{x}}\right)$$
$$= \log_5 \sqrt{x}$$

Bonus Question. *(3 marks)*
Find the radius of the circle with center $(4,-3)$ and tangent to the line $y = 2x - 1$



normal to the line and passes through (4,-3)

$y = -\frac{1}{2}x + b$

sub $(4,-3)$

$-3 = -\frac{1}{2}(4) + b$

$-3 = -2 + b$

$-1 = b$

$\therefore\ y = -\frac{1}{2}x - 1$

Intersection of tangent and normal

① $y = 2x - 1$

② $y = -\frac{1}{2}x - 1$

$2x - 1 = -\frac{1}{2}x - 1$

$x = 0$

if $x = 0$, $y = 2(0) - 1 = -1$

$r = d = \sqrt{(0-4)^2 + (-1-(-3))^2}$ distance between $(0,-1)$ and $(4,-3)$

$= \sqrt{20}$