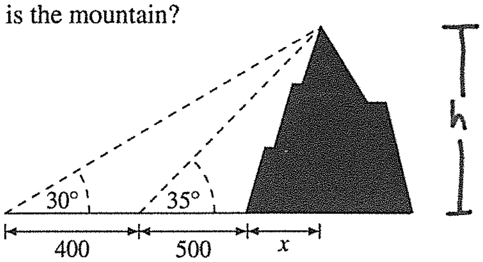
Name: Y. Lamontagne
Student ID: ______________

Test 3

This test is graded out of 50 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page. This test consists of 9 questions and one bonus question. The maximum possible grade is 50/50. Please ensure that you have a complete test. This test must be returned intact.

Question 1.[1] *(5 marks)* A person standing 500 ft from the base of a mountain measures the angle of elevation from the ground to the top of the mountain to be 35°. The person then walks 400 ft straight back and measures the angle of elevation to now be 30°. How tall is the mountain?



$$\begin{cases} \tan 35^\circ = \dfrac{h}{500+x} \\ \tan 30^\circ = \dfrac{h}{900+x} \end{cases}$$

$$\begin{cases} (500+x)\tan 35^\circ = h & (1) \\ (900+x)\tan 30^\circ = h & (2) \end{cases}$$

$(1) = (2)$

$$(500+x)\tan 35^\circ = (900+x)\tan 30^\circ$$
$$500\tan 35^\circ + x\tan 35^\circ = 900\tan 30^\circ + x\tan 30^\circ$$
$$500\tan 35^\circ - 900\tan 30^\circ = x\tan 30^\circ - x\tan 35^\circ$$
$$500\tan 35^\circ - 900\tan 30^\circ = x(\tan 30^\circ - \tan 35^\circ)$$

$$x = \frac{500\tan 35^\circ - 900\tan 30^\circ}{\tan 30^\circ - \tan 35^\circ} = 1380 \text{ ft}$$

$$\therefore h = (500 + 1380)\tan 35^\circ = 1316 \text{ ft.}$$

Question 2. *(3 marks)* What is the linear velocity in meters per seconds of the tip of a consaw blade spinning at 5200 rpm with a blade of 16 inches in diameter? Note: 1 rpm = 0.10472 rad/s

$$V = \omega r = 544.544(0.2032) = 110 \text{ m/s}$$

$$r = \frac{d}{2} = \frac{16}{2} = 8 \text{ in} = 8 \text{ in} \cdot \frac{2.54 \text{ cm}}{1 \text{ in}} = 20.32 \text{ cm} = 0.2032 \text{ m}$$

$$\omega = 5200 \text{ rpm} \cdot \frac{0.10472 \text{ rad/s}}{1 \text{ rpm}} = 544.544 \text{ rad/s}$$

[1] Modified from Trigonometry by Michael Corral

Question 3.

a. *(4 marks)* Prove the following trigonometric identity. SHOW ALL YOUR WORK

$$\frac{\sin(2\theta)}{\cos\theta}+\frac{\cos(2\theta)}{\sin\theta}\stackrel{?}{=}\csc\theta$$

$$\csc\theta \stackrel{?}{=} \frac{2\sin\theta\cancel{\cos\theta}}{\cancel{\cos\theta}} + \frac{2\cos^2\theta-1}{\sin\theta}$$

$$\csc\theta \stackrel{?}{=} 2\sin\theta + \frac{2\cos^2\theta-1}{\sin\theta}$$

$$\csc\theta \stackrel{?}{=} \frac{2\sin^2\theta}{\sin\theta} + \frac{2\cos^2\theta-1}{\sin\theta}$$

$$\csc\theta \stackrel{?}{=} \frac{2\sin^2\theta+2\cos^2\theta-1}{\sin\theta}$$

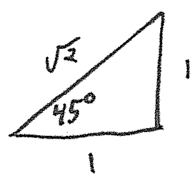
$$\csc\theta \stackrel{?}{=} \frac{2(\sin^2\theta+\cos^2\theta)-1}{\sin\theta}$$

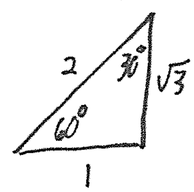
$$\csc\theta \stackrel{?}{=} \frac{2(1)-1}{\sin\theta}$$

$$\csc\theta = \frac{1}{\sin\theta}$$

b. *(4 marks)* Use $105° = 60° + 45°$ to find the exact value of $\csc(105°)$. SHOW ALL YOUR WORK

$$\sin(105) = \sin(60°+45°) = \sin(60°)\cos(45°) + \sin(45°)\cos(60°)$$





$$= \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\cdot\frac{1}{2}$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}}$$

$$\csc(105°) = \frac{1}{\sin(105°)} = \frac{1}{\frac{\sqrt{3}+1}{2\sqrt{2}}} = \frac{2\sqrt{2}}{\sqrt{3}+1}$$

Question 4.

a. *(1 mark)* Convert the angle 330° to radians. Give the exact value.

b. *(1 mark)* Convert the angle $\frac{11\pi}{15}$ to degrees.

c. *(2 mark)* Find an angle θ, such that $0° \leq \theta < 360°$, which is coterminal with 934°. In which quadrant does θ lie?

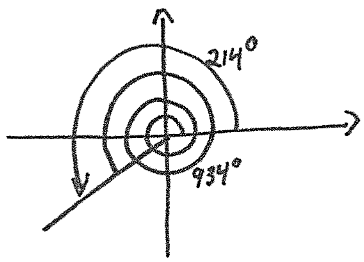
d. *(2 mark)* Find the exact value of $\sin(240°)$

e. *(2 mark)* Find the exact value of $\tan(\frac{4\pi}{3})$

a) $330°\left(\frac{\pi}{180°}\right) = \frac{11\pi}{6}$

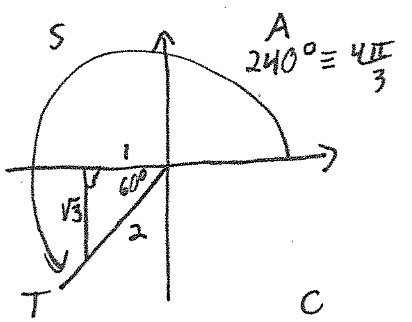
b) $\frac{11\cancel{\pi}}{15} \cdot \frac{180°}{\cancel{\pi}} = 132°$

c)



$\theta = 214°$ and lies in the 3rd quadrant.

d)



$\sin(240°) = -\frac{\sqrt{3}}{2}$

e) $\tan\left(\frac{4\pi}{3}\right) = \frac{1}{\sqrt{3}}$

Question 5. *(5 marks)* Find the amplitude, period, and phase shift *(displacement)*. Then graph one period of the given function.

$y = -2\cos(3x + \pi)$

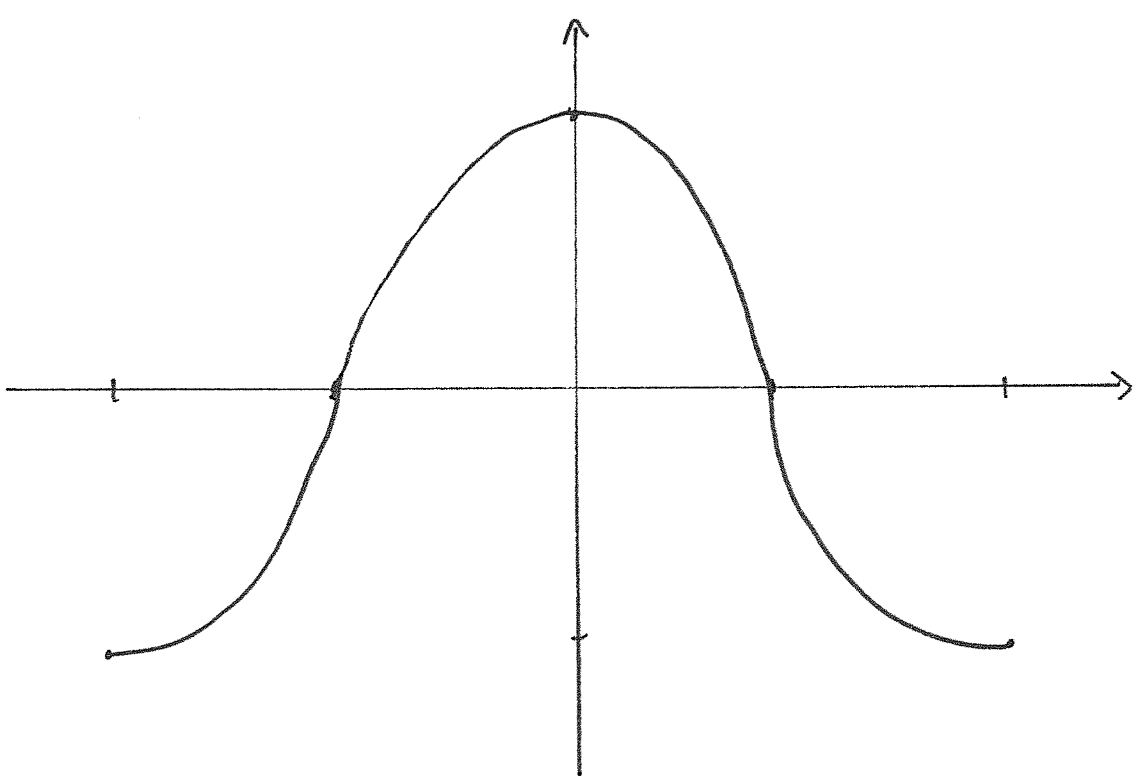
amplitude $= |a| = |-2| = 2$

period $= \frac{2\pi}{b} = \frac{2\pi}{3}$

phase shift $= \frac{-c}{b} = \frac{-\pi}{3}$

Key values:

x = phase shift + $\frac{\text{period}}{4}$ x multiple	y
$\frac{-\pi}{3} + \frac{\pi}{6}(0) = \frac{-\pi}{3}$	-2
$-\frac{\pi}{3} + \frac{\pi}{6}(1) = \frac{-\pi}{6}$	0
$-\frac{\pi}{3} + \frac{\pi}{6}(2) = 0$	2
$-\frac{\pi}{3} + \frac{\pi}{6}(3) = \frac{\pi}{6}$	0
$-\frac{\pi}{3} + \frac{\pi}{6}(4) = \frac{\pi}{3}$	-2



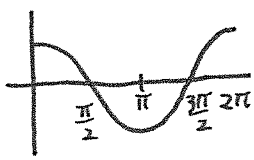
Question 6. *(5 marks)* Solve the following equation in the interval $[0, 2\pi)$.

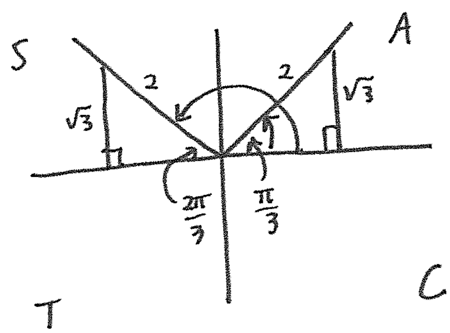
$$2\sin x\cos x - \sqrt{3}\cos x = 0$$

$$\cos x\,(2\sin x - \sqrt{3}) = 0$$

$\cos x = 0$

$\sin x = \frac{\sqrt{3}}{2}$





$x = \frac{\pi}{2}, \frac{3\pi}{2}$

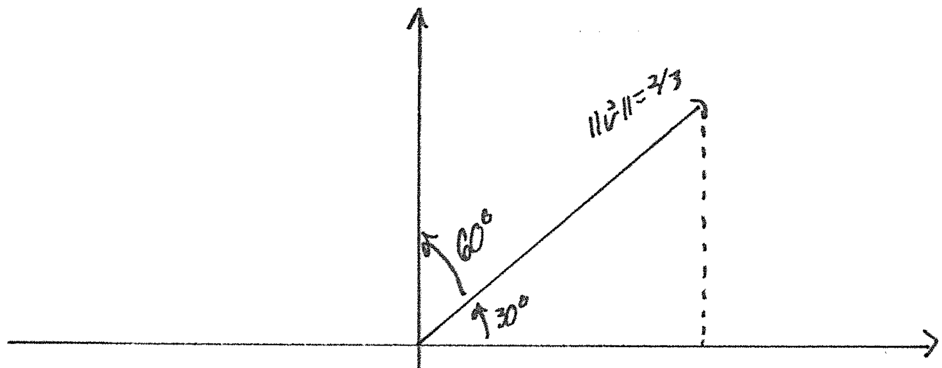
$x = \frac{\pi}{3}, \frac{2\pi}{3}$

Question 7.[2]

a. *(2 marks)* Find the component form of the vector $\vec{v}$ with magnitude $||\vec{v}|| = \frac{2}{3}$ which, when in standard position, lies in Quadrant 1 and makes a $60°$ angle with the positive y-axis. Give exact values.

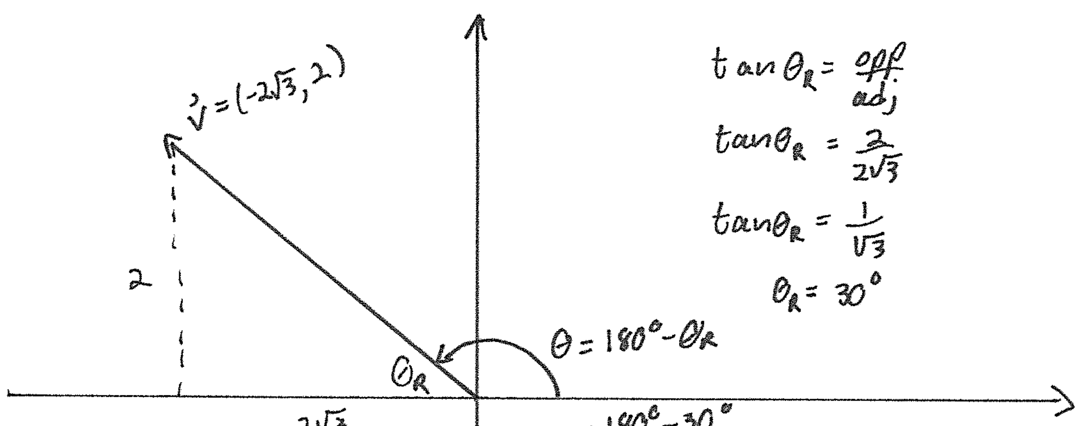
b. *(2 marks)* For the vector $\vec{v} = (-2\sqrt{3}, 2)$, find the magnitude $||\vec{v}||$ and an angle θ, with $0° \leq \theta < 360°$, such that $\vec{v} = ||\vec{v}||(\cos(\theta), \sin(\theta))$.

a)



$$\vec{v} = \left(\tfrac{2}{3}\cos 30°, \tfrac{2}{3}\sin 30°\right)$$
$$= \left(\tfrac{\cancel{2}}{3}\tfrac{\sqrt{3}}{\cancel{2}}, \tfrac{\cancel{2}}{3}\tfrac{1}{\cancel{2}}\right)$$
$$= \left(\tfrac{\sqrt{3}}{3}, \tfrac{1}{3}\right)$$

b)



$$\tan\theta_R = \frac{opp}{adj}$$
$$\tan\theta_R = \frac{2}{2\sqrt{3}}$$
$$\tan\theta_R = \frac{1}{\sqrt{3}}$$
$$\theta_R = 30°$$

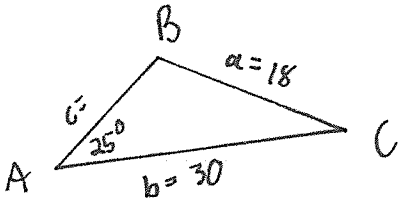
$$\theta = 180° - \theta_R$$
$$= 180° - 30°$$
$$= 150°$$

$$||\vec{v}|| = \sqrt{(-2\sqrt{3})^2 + 2^2} = \sqrt{4\cdot 3 + 4}$$
$$= \sqrt{16}$$
$$= 4$$

$$\vec{v} = 4(\cos 150°, \sin 150°)$$

[2]Modified from Precalculus by Carl Stitz and Jeff Zeage

Question 8. *(6 marks)* Solve the triangle ΔABC where $a = 18$, $A = 25°$, $b = 30$. Then find the area of the triangle. Note: If there is more than one possibility, give both solutions.



$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 25°}{18} = \frac{\sin B}{30}$$

$$\frac{5}{3}\sin 25° = \sin B$$

$$135.22° \text{ or } 44.78° = B$$

If $B = 44.78°$

then $C = 180° - 44.78° - 25° = 110.22°$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{18}{\sin 25°} = \frac{c}{\sin 110.22°}$$

$$c = 39.97$$

$$\text{Area} = \frac{1}{2} ab \sin C$$

$$= \frac{1}{2}(18)(30)\sin 110.22$$

$$= 253.36\ u^2$$

If $B = 135.22°$

then $C = 180° - 135.22° - 25° = 19.78°$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{18}{\sin 25°} = \frac{c}{\sin 19.78°}$$

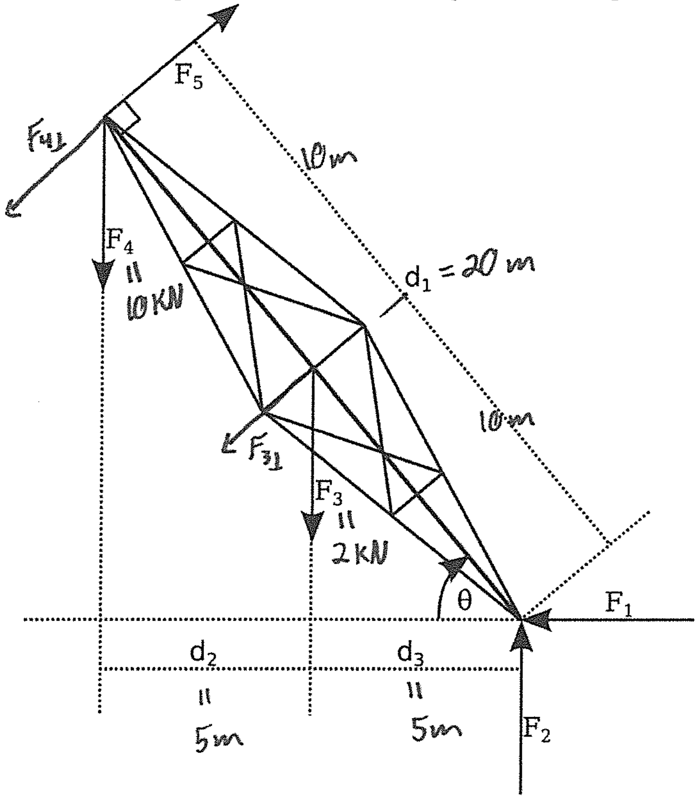
$$c = 14.41$$

$$\text{Area} = \frac{1}{2} ab \sin C$$

$$= \frac{1}{2}\, 18(30)\sin 19.78°$$

$$= 91.37\ u^2$$

Question 9. (6 marks) Given that $F_3 = 2.00\ kN$, $F_4 = 10.00\ kN$, $d_1 = 20.00\ m$, $d_2 = 5.00\ m$, $d_3 = 5.00\ m$ and that the crane is in mechanical equilibrium, find the missing forces and angle.



$$\cos\theta = \frac{5+5}{20} = \frac{1}{2}$$

$$\theta = 60^\circ$$

The crane is in mechanical equilibrium iff

$$\sum F_i = 0$$

and

$$\sum M_i = 0 .$$

$$\sum F_i = 0 \text{ iff}$$

$$F_2 - F_3 - F_4 + F_{5y} = 0$$

and

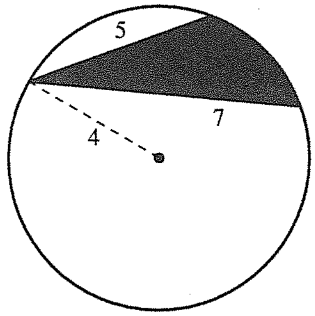
$$F_{5x} - F_1 = 0$$
$$F_1 = F_{5x}$$
$$F_1 = F_5 \cos 30^\circ$$
$$F_1 = 5.5 \frac{\sqrt{3}}{2}$$
$$= 4.76\ KN$$

$$\sum M_i = 0 \text{ iff}$$

$$CWM = CCWM$$
$$F_5 (20) = F_{4\perp}(20) + F_{3\perp}(10)$$
$$F_5(20) = 10\cos 60^\circ (20) + 2\cos 60^\circ (10)$$
$$F_5 = \frac{10(\frac{1}{2})(20) + 2(\frac{1}{2})(10)}{20}$$
$$= 5.5\ KN$$

$$F_2 - 2 - 10 + F_5 \sin 30^\circ = 0$$
$$F_2 = 2 + 10 - 5.5 \sin 30^\circ$$
$$F_2 = 9.25\ KN$$

Bonus Question. *(3 marks)*
Find the area of the shaded region in the following figure.



$AB^2 = OA^2 + OB^2 - 2OAOB\cos\angle AOB$

$5^2 = 4^2 + 4^2 - 2(4)(4)\cos\angle AOB$

$\cos\angle AOB = 0.21875$

$\angle AOB = 77.36^\circ$

$AC^2 = OA^2 + OC^2 - 2OAOC\cos\angle AOC$

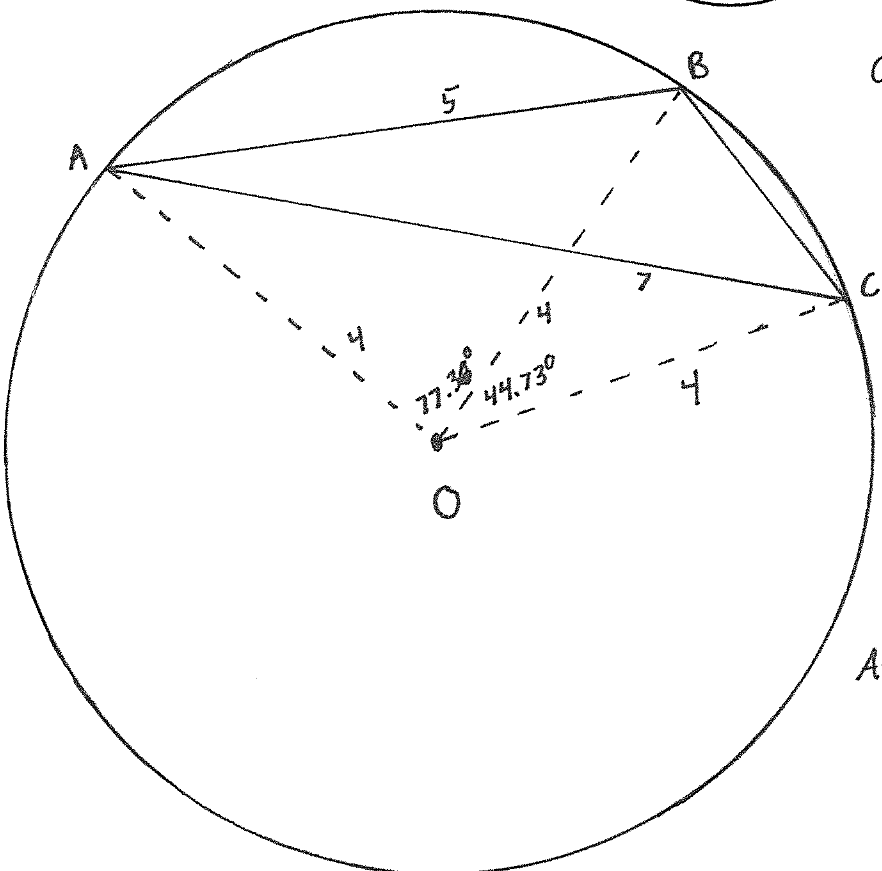
$7^2 = 4^2 + 4^2 - 2(4)(4)\cos\angle AOC$

$\cos\angle AOC = -0.53125$

$\angle AOC = 122.09^\circ$

Then $\angle BOC = 122.09^\circ - 77.36^\circ$

$= 44.73^\circ$



$BC^2 = OB^2 + OC^2 - 2OBOC\cos\angle BOC$

$BC^2 = 4^2 + 4^2 - 2(4)(4)\cos 44.73^\circ$

$BC = 3.04$

Let $s = \frac{1}{2}(5 + 7 + 3.04)$

$= 7.7$

Area $\triangle ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{7.7(7.7-5)(7.7-7)(7.7-3.04)}$

$= 8.24$

Area of sector $OBC = \frac{1}{2}\theta r^2$

$= \frac{1}{2} 44.73\left(\frac{\pi}{180}\right)(4)^2$

$= 5.964$

Area $\triangle OBC = \frac{1}{2}(4)(4)\sin 44.73^\circ = 5.63$

Area $=$ area $\triangle ABC$ + area of sector OBC − area $\triangle OBC$

$= 8.24 + 5.964 - 5.63$

$= 8.57$