In-class Assignment #6 – Normal Distributions
Dawson College - Introduction to Statistics (201-922-DW)
Instructor: Émilie Richer
(worth 2% of final grade)
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Name: __SOLUTIONS__

Question 1
Given a standard normal distribution, find the area under the curve that lies
- (a) to the left of $z = -1.39$;
- (b) to the right of $z = 1.96$;
- (c) between $z = -2.16$ and $z = -0.65$;
- (d) to the left of $z = 1.43$;
- (e) to the right of $z = -0.89$;
- (f) between $z = -0.48$ and $z = 1.74$.

Question 2
Given a normal distribution with $\mu = 30$ and $\sigma = 6$, find
- (a) the normal curve area to the right of $x = 17$;
- (b) the normal curve area to the left of $x = 22$;
- (c) the normal curve area between $x = 32$ and $x = 41$;
- (d) the value of x that has 80% of the normal curve area to the left;
- (e) the two values of x that contain the middle 75% of the normal curve area.

Question 3
Given the normally distributed variable X with mean 18 and standard deviation 2.5, find
- (a) $P(X < 15)$;
- (b) the value of k such that $P(X < k) = 0.2236$;
- (c) the value of k such that $P(X > k) = 0.1814$;
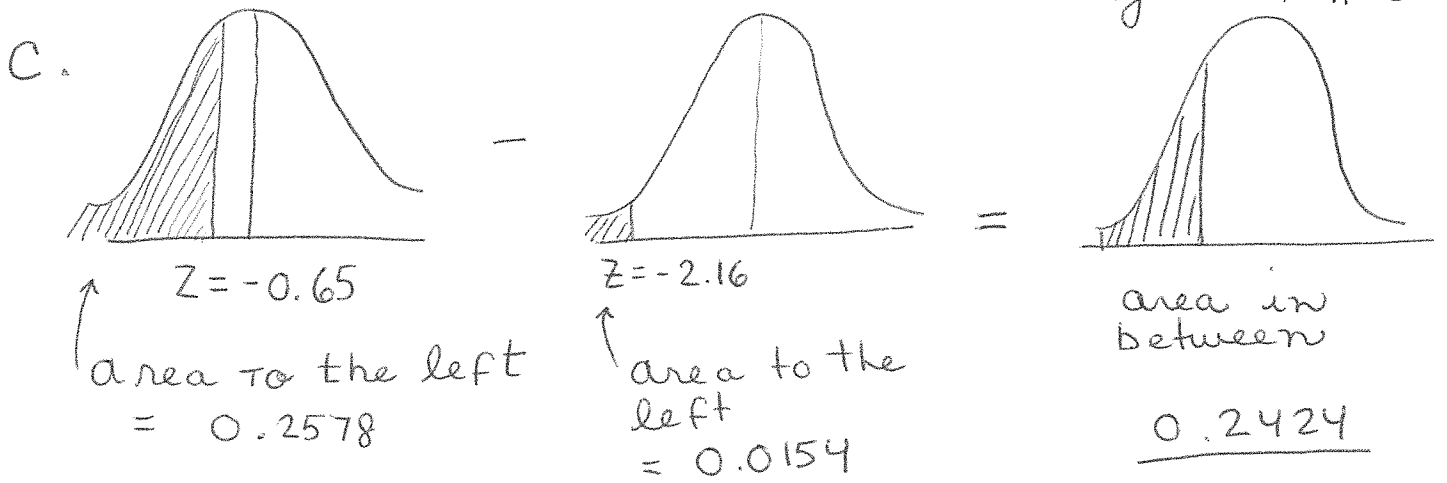- (d) $P(17 < X < 21)$.

Solutions (Q1)

We use the standard Normal Table directly

① Values in the Z-table give areas to the left of z.

a. __0.0823__

b. TO THE LEFT OF Z = 1.96
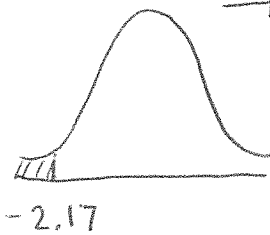 is 0.9750
 THEREFORE area to the right is __0.0250__

C.

Z = -0.65

area to the left
= 0.2578

Z = -2.16

area to the
left
= 0.0154

area in
between

0.2424

d. $\underline{0.9236}$

e. $\underline{0.8133}$

f. $0.9591 - 0.3156 = \underline{0.6435}$

$\underline{\text{QUESTION 2}}$

$\mu = 30$ & $\sigma = 6$

a. $X = 17$ $\quad Z = \dfrac{17-30}{6} = -2.17$

-2.17

area
to the
left 0.015

So area
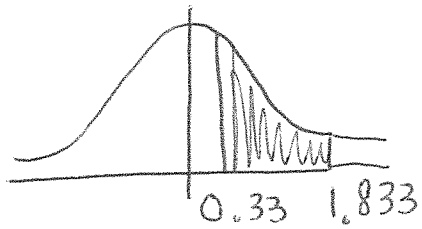to the right $= 1 - 0.015$
$= \underline{0.985}$

b. $X = 22 \longrightarrow Z = \dfrac{22-30}{6} = -1.33$

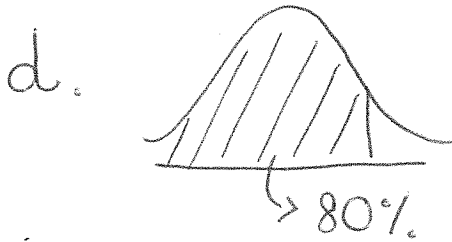area to the left $= \underline{0.0918}$

C. Between $X = 32$ & $X = 41$

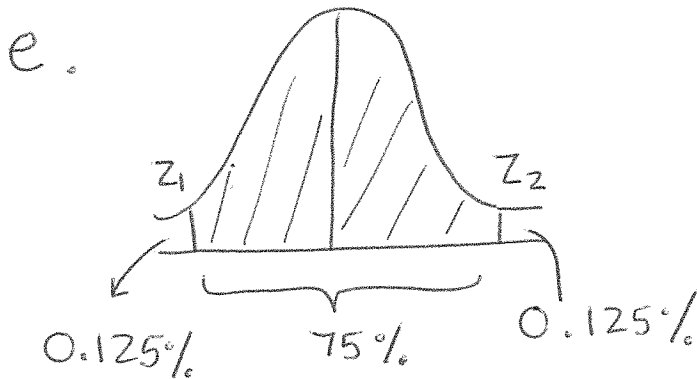$Z = \dfrac{32-30}{6} = 0.33$

$\longrightarrow Z = \dfrac{41-30}{6} = 1.833$

$$= 0.9664 - 0.6293$$
$$= 0.3371$$

d.

$> 80\%$

$$Z = 0.84$$

$$0.84 = \frac{X - 30}{6} \qquad X = 35.04$$

e.

Z_1 Z_2

0.125% 75% 0.125%

$$Z_1 \rightarrow 0.125 \text{ to the left}$$
$$Z_2 \rightarrow 0.875 \text{ to the left}$$

$$Z_1 = \qquad -1.15$$
$$Z_2 = \qquad +1.15$$

$$-1.15 = \frac{X_1 - 30}{6} \qquad\qquad 1.15 = \frac{X_2 - 30}{6}$$

$$X_1 = 23.1 \qquad\qquad X_2 = 36.9$$

Question 3

$\mu = 18 \qquad \sigma = 2.5$

a. $P(X < 15) = P(Z < -1.2) = 0.1151$

$$Z = \frac{15 - 18}{2.5} = -1.2$$

b.
$$P(X < K) = 0.2236$$
$$P(Z < -0.76) = 0.2236$$
$$-0.76 = \frac{X - 18}{2.5} \qquad \underline{X = 16.1}$$
$$P(X < 16.1) = 0.2236$$

c.
$$P(X > K) = 0.1814$$
$$P(X < K) = 1 - 0.1814$$
$$= 0.8186$$
$$P(Z < 0.91) = 0.8186$$
$$0.91 = \frac{X - 18}{2.5} \qquad \underline{X = 20.275}$$
$$P(X > 20.275) = 0.1814$$

d.
$$P(17 < X < 21)$$
$$Z_1 = \frac{17 - 18}{2.5} = -0.4 \qquad Z_2 = \frac{21 - 18}{2.5} = 1.2$$
$$P(-0.4 < Z < 1.2) = 0.8849 - 0.3446$$
$$= \underline{0.5403}$$