

Test 1

This test is graded out of 35 marks. No books, notes or cell phones are allowed. You must show all your work

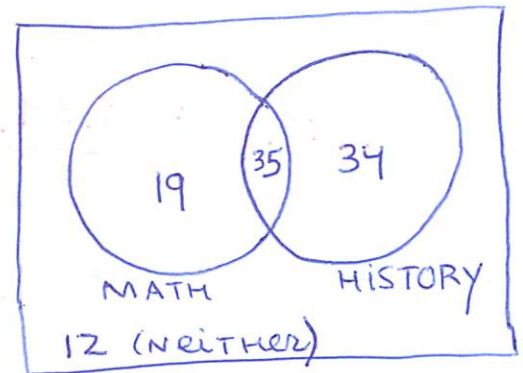
Question 1. (5 marks)

In a highschool graduating class of 100 students, 54 studied mathematics, 69 studied history, and 35 studied both mathematics and history. If one of these students is selected at random, find the probability that (use a Venn diagram if you would like)

a. the student took mathematics or history;

$$\begin{aligned} &54 \text{ MATHEMATICS} \\ &54 - 35 = 19 \text{ ONLY MATHEMATICS} \\ &69 \text{ HISTORY} \\ &69 - 35 = 34 \text{ ONLY HISTORY} \end{aligned}$$

$$\begin{aligned} P(\text{MATH} \cup \text{HIST}) &= P(\text{MATH}) + P(\text{HIST}) - P(\text{MNH}) \\ &= \frac{54}{100} + \frac{69}{100} - \frac{35}{100} = \frac{88}{100} = \underline{0.88} \end{aligned}$$



b. the student did not take either of these subjects;

$$1 - 0.88 = \underline{0.12}$$

c. the student took history but not mathematics.

$$\frac{34}{100} = \underline{0.34}$$

Question 2. (10 marks)

Find the number of different ways of:

- a. forming a 5-player hockey team from a group of 10 people;

$${}_{10}C_5 = \underline{252}$$

- b. seating 10 people in a row of 12 seats;

$${}_{12}P_{10} = \underline{239,500,800}$$

- c. selecting a lunch consisting of a soup, sandwich, dessert and drink if there are 3 choices of soup, 4 choices of sandwich, 2 choices of dessert and 5 choices of drink.

$$3 \times 4 \times 2 \times 5 = \underline{120}$$

- d. ordering the letters in the word SUCCESSFULLY

$$\frac{12!}{3!2!2!2!} \leftarrow \begin{array}{l} \text{TOTAL \# OF letters} \\ \text{repeated letters} \end{array}$$

$$= \underline{9,979,200}$$

- e. selecting a 5-card hand including exactly two aces from a standard deck of 52 cards

$${}_{48}C_3 {}_4C_2 = \underline{103,776}$$

Question 3. (5 marks)

The probability that a doctor correctly diagnoses a particular illness is 0.7. Given that the doctor makes an incorrect diagnosis, the probability that the patient files a lawsuit is 0.9. What is the probability that the doctor makes an incorrect diagnosis and the patient files a lawsuit.

C = correct diagnosis
I = incorrect diagnosis
L = files a lawsuit

$$P(I) = 0.3 \quad P(L|I) = 0.9$$

we want $P(L \cap I) = P(L|I) \cdot P(I)$
 $= (0.9) \cdot (0.3)$
 $= \underline{0.27}$

Question 4. (5 marks)

Calculate the sample mean \bar{x} and the sample standard deviation s of the following sample of final grades from last year's statistics class:

84 92 35 61 71 89 95 56 78 76
82 62 44 55 78 85 86 21 98 79

Using a calculator in STAT MODE is the FASTEST WAY TO PROCEED

$$\bar{x} = \frac{\sum_{i=1}^{20} x_i}{20} = \frac{[84 + 92 + \dots + 98 + 79]}{20} = \underline{71.35}$$

$$s = \sqrt{\frac{\sum_{i=1}^{20} (71.35 - x_i)^2}{n-1}} = \underline{20.71}$$

Question 5. (5 marks)

A student guesses at each of the 5 questions on a multiple choice quiz. If each question has 4 choices (1 correct, 3 wrong), find the probability that the student passes the quiz.

$$P(\text{passing}) = P(3 \text{ correct}) + P(4 \text{ correct}) + P(5 \text{ correct})$$

$$= \frac{{}^5C_3 3^2}{4^5} + \frac{{}^5C_4 \cdot 3}{4^5} + \frac{{}^5C_5}{4^5}$$

3 CHOICES OF WRONG ANSWERS FOR 2 QUESTIONS

OF WAYS OF ANSWERING 3 CORRECT QUESTIONS FROM 5

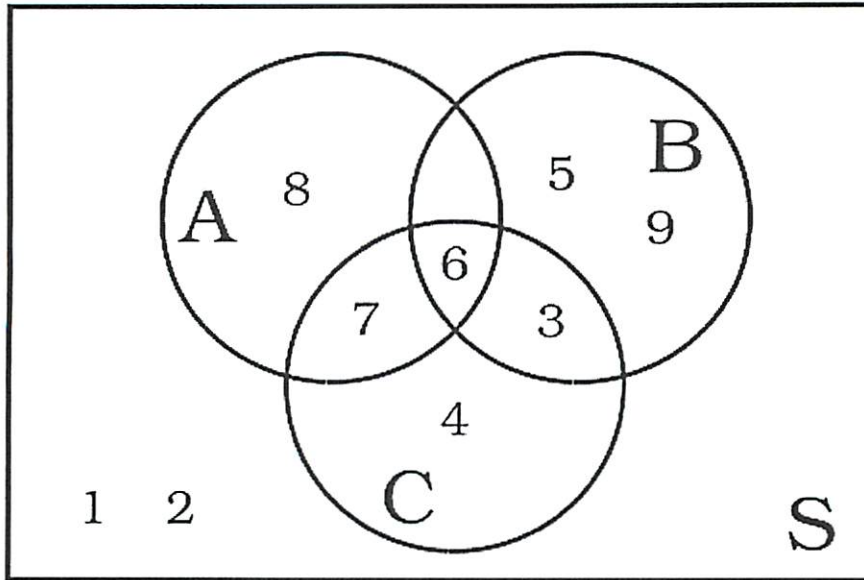
OF WAYS OF ANSWERING QUIZ

$$= \frac{90}{1024} + \frac{45}{1024} + \frac{1}{1024} = \frac{136}{1024} = \underline{0.1035}$$

$$P(\text{passing}) = \frac{{}^5C_3 3^2 + {}^5C_4 \cdot 3 + {}^5C_5}{4^5} = \frac{90 + 45 + 1}{1024} = \frac{136}{1024} = 0.1035$$

Question 6. (5 marks)

Consider the following Venn diagram illustrating events A, B and C in the sample space $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.



List the elements of the following events.

$$A \cap C = \{6, 7\}$$

$$B \cup C = \{3, 4, 5, 6, 7, 9\}$$

$$(A \cup C)' = \{1, 2, 5, 9\}$$

$$A \cap B \cap C = \{6\}$$

$$(A \cap C) \cup B = \{3, 5, 6, 7, 9\}$$