NAME: SOLUTIONS

Introduction to Statistics - Test 2

Friday November 5th 2015

PART 1 MULTIPLE CHOICE QUESTIONS (20 MARKS)

Questions 1-10 are multiple choice questions worth two marks each. Clearly circle your answer, if there is any ambiguity in your response (for example two selected answers) you will not receive any marks for the question.

1. The random variable X has the following probability distribution

X	1	2	5	10
P(X)	0.2	0.1	0.4	

What is the value of P(X=10)?

(a) 0.2
(b) 0.3
(c) 0.5
(d) 0.1
(e) None of the above

a probability distribution has the property that $\sum_x P(x) = 1$

So $P(10) = 0.3$

2. Referring to the random variable of question 1, the mean and standard deviation of X are, respectively:

(a) 4.5 and 16.75
(b) 5.4 and 11.44
(c) 5.4 and 3.38
(d) 4.5 and 4.09
(e) None of the above

$$\mu = \sum_x x P(x) = 1(0.2) + 2(0.1) + 5(0.4) + 10(0.3) = 5.4$$

$$\sigma^2 = \sum_x (x-\mu)^2 P(x) = (1-5.4)^2 0.2 + (2-5.4)^2(0.1) + (5-5.4)^2 0.4 + (10-5.4)^2(0.3)$$
$$= 3.872 + 1.156 + 0.064 + 6.348 = 11.44$$

So $\sigma = \sqrt{11.44} = 3.38$

3. Again, referring to the random variable of question 1, select the value of P(X>1):

(a) 1
(b) 0.6
(c) 0.8
(d) 0.7
(e) None of the above

$$P(x>1) = P(x=2) + P(x=5) + P(x=10)$$
$$= 0.1 + 0.4 + 0.3$$
$$= 0.8$$

4. Which of the following statements is not an assumption of the binomial experiment?

(a) All trials are identical and independent
(b) Each trial's outcome is classified as a success or a failure
(c) The number of successes in the trials is counted
(d) The probability of success is equal to 0.5 in all trials
(e) All above statements are assumptions of the binomial experiment

5. A multiple choice test has 20 questions with each question having 4 possible solutions (1 correct, 3 incorrect). A student randomly guesses at each of the 20 questions. Let X denote the # of correct answers selected, what is the value of $\mu = E(X)$?

(a) 15
(b) 5
(c) 10
(d) 3.75
(e) None of the above

BINOMIAL $n = 20$ $p = 1/4 = 0.25$

$\mu = np$

$= 20(0.25) = 5$

6. Using the data of question 5, what is the variance σ^2 of X?

(a) 5
(b) 2.24
(c) 3.75
(d) 1.94
(e) None of the above

$\sigma^2 = npq = 20(0.25)(0.75)$

$= 3.75$

7. Using the data of question 5, what is the formula that best describes the probability P(X=8)?

(a) $P(X=8) = {}_{20}C_8(0.25)^8(0.75)^{12}$
(b) $P(X=8) = {}_{20}C_8(0.25)^{12}(0.75)^8$
(c) $P(X=8) = 8(0.25) + 12(0.75)$
(d) $P(X=8) = {}_{20}C_{12}(0.25)^{12}(0.75)^8$
(e) None of the above

8. Which of the following statements is <u>not true</u> about the normal distribution?

(a) The curve approaches the x-axis gradually on either side of the mean
(b) The mean, median and mode are the same value
(c) The curve is bell-shaped
(d) The curve is symmetrical with respect to the vertical line drawn from the peak of the curve
(e) 95% of the area beneath the curve is found within 1 standard deviation from the mean

9. If $\mu = 5\,000$ and $\sigma = 50$ the z-value of $X = 5\,025$ is?

(a) -1
(b) 1
(c) 0.5
(d) -0.5
(e) None of the above

$Z = \frac{X - \mu}{\sigma} = \frac{5025 - 5000}{50} = 0.5$

10. Which of the following is <u>not</u> correct about a standard normal distribution?

(a) $P(Z \leq 1.5) = 0.9332$
(b) $P(Z \geq -2.5) = 0.4938$
(d) $P(0 \leq Z \leq 1.5) = 0.4332$
(d) $P(Z \leq -1.0) = 0.1587$
(e) $P(Z \geq 2.0) = 0.0228$

a. correct

b. $1 - P(Z < -2.5) = 1 - 0.0062$

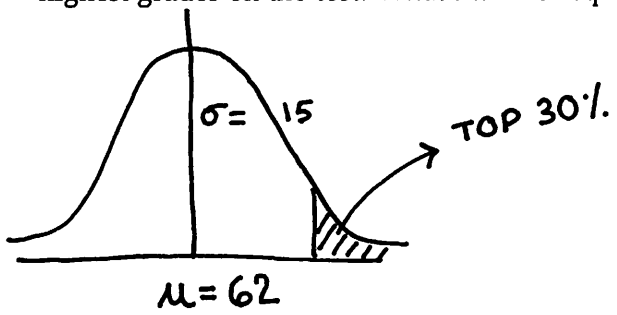
$= 0.9938$

b is FALSE

PART 2 LONG ANSWER QUESTIONS (20 MARKS)

Questions 11-15 are long answer questions in which **you must show all your work** in order to receive full marks.

11. (5 marks) The marks on a statistics test are normally distributed with a mean μ of 62 and a variance σ^2 of 225. The teacher wishes to assign a letter grade of B or higher to the students with the top 30% highest grades on the test. What mark is required for a student to get a grade of B or higher?



$$Z = \frac{X-62}{15}$$

FIND z VALUE WITH AREA CLOSEST TO 0.7; $Z = 0.52$

$$0.52 = \frac{X-62}{15}$$

$$X = 69.8$$

STUDENTS NEED A MARK OF 69.8 OR MORE IN ORDER TO GET A B OR HIGHER.

12. (5 marks) When a pair of dice is rolled, 36 equally likely outcomes are possible. Let X denote the larger of the values showing in the dice. If both dice come up the same then X denotes the common value.

a. List the 36 equally possible outcomes for the pair of dice

1 1	21	31	41	51	61
1 2	22	32	42	52	62
1 3	23	33	43	53	63
1 4	24	34	44	54	64
1 5	25	35	45	55	65
1 6	26	36	46	56	66

b. List the possible values of the random variable X.

1, 2, 3, 4, 5, 6

c. Give the probability distribution of the random variable X.

X	1	2	3	4	5	6
P(X)	$1/36$	$3/36$	$5/36$	$7/36$	$9/36$	$11/36$

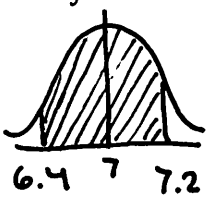
d. Determine the expected value μ of X.

$$\mu = \sum_x x\,P(x) = 1\left(\frac{1}{36}\right) + 2\left(\frac{3}{36}\right) + 3\left(\frac{5}{36}\right) + 4\left(\frac{7}{36}\right) + 5\left(\frac{9}{36}\right) + 6\left(\frac{11}{36}\right)$$

$$= \underline{4.472}$$

13. (6 marks) The life of a laboratory microscope is 7 years with a standard deviation of 1 year. Assuming that the lives of these microscopes follow a normal distribution find

a. the probability that that the life of a laboratory microscope selected at random falls between 6.4 and 7.2 years



$$P(6.4 \leq X \leq 7.2) = P\left(\frac{6.4-7}{1} \leq z \leq \frac{7.2-7}{1}\right)$$
$$= P(-0.6 \leq z \leq 0.2)$$
$$= 0.5793 - 0.2743$$
$$= 0.305$$

b. the probability that that the mean life of 9 laboratory microscopes selected at random falls between 6.4 and 7.2 years

$$\mu_{\bar{x}} = 7 \quad \sigma_{\bar{x}} = 1/\sqrt{9} = 1/3$$

$$P(6.4 \leq \bar{x} \leq 7.2) = P\left(\frac{6.4-7}{1/3} \leq \bar{x} \leq \frac{7.2-7}{1/3}\right)$$
$$= P(-1.8 \leq \bar{x} \leq 0.6)$$
$$= 0.7258 - 0.0359 = 0.6899$$

c. Did you expect to get a larger or a smaller value in b. than in a. Explain.

We expect a larger value because the standard deviation is smaller, this means that more data will be concentrated about the mean.

Therefore the area between 6.4 & 7.2 will grow.

14. (4 marks) The probability that a patient recovers from a rare blood disease is 0.4. If 100 people are known to have contracted this disease, approximate the probability that fewer than 30 will survive.

BINOMIAL $n = 100$

$p = 0.4$

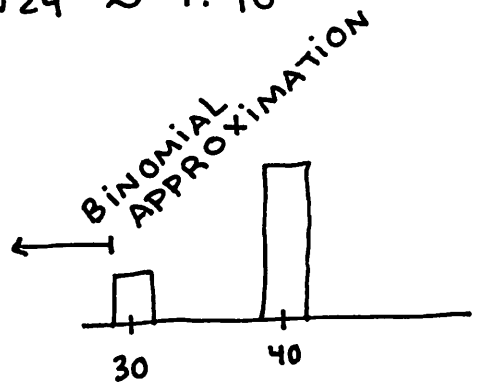
$q = 0.6$

$\mu = np = 40$

$\sigma = \sqrt{npq} = \sqrt{100(0.4)(0.6)}$

$= \sqrt{24} \approx 4.90$

WE WANT $P(X < 30)$



APPROXIMATION USING NORMAL b/c np & $nq > 5$

$P(X < 29.5)$

$= P\left(Z < \frac{29.5 - 40}{4.9}\right)$

$= P(Z < -2.14)$

$= 0.0162$

THE PROBABILITY IS 0.0162 THAT FEWER THAN 30 PEOPLE WILL SURVIVE.