Student Name: ____________________
Student ID: ____________________

Comprehensive Examination (CE): Logic (Oral Examination)

Text: Proofs and Concepts: the fundamentals of abstract mathematics by Dave Witte Morris and Joy Morris.
`http://people.uleth.ca/~dave.morris/books/proofs+concepts.pdf`

Reading: Chapters 1 to 3.

Evaluation: The CE is associated to the linear algebra science course (201-NYC-05). Note that if the student fails the CE, the student cannot graduate. The CE mark for 201-NYC-05 will be either a pass or a fail. To pass the CE the student must obtain 60% or more on the oral examination.

Sample Oral Examination: See back of page.

Oral Examination Date and Location:

Consequence to Missing the Oral Examination Date: Failure of the CE unless a valid medical note is provided.

I hereby acknowledge that I have read, understand and agree to the terms of this Comprehensive Evaluation(CE).

Student Signature: ____________________

Date: ____________________

Sample Oral Examination:

Question 1. Given the following symbolization key:
A: Alexander Berkman loves Emma Goldman
B_1: Alexander Berkman buys bread.
B_2: Emma Golman buys bread.
E: Emma Goldman loves Alexander Berkman.
F_1: Alexander Berkman buys flowers.
F_2: Emma Goldman buys flowers.
P_1: Alexander Berkman protests.
P_2: Emma Goldman protests.

Translate each English language statement into Propositional Logic.

a. *(1 mark)* Emma buys flowers and Aleaxander buys bread if, neither Alexander loves Emma nor Emma loves Alexander.

Translate each Propositional Logic statement into English.

b. *(1 mark)* $(\neg P_2 \wedge B_2) \iff E$

Question 2. *(2 marks)* Determine wether the following statement is a tautology, contradiction, or contingent statement. Justify your conclusion.

$$[(\neg A \to B) \wedge \neg B] \to A$$

Question 3. *(2 marks)* Determine whether the following is a valid argument. Justify your conclusion.

$$(\neg P_2 \wedge B_2) \iff E \therefore E$$

Question 4. (3 marks) Is the following is possible? If it is possible, give an example. If it is not possible, explain why.

An invalid argument, the conclusion of which is a tautology.

Question 5.

a. *(0.5 mark)* Translate the English statement into a propositional logic statement:
Emma Goldman does not love Alexander Berkman if Alexander does not buy flowers.

b. *(0.5 mark)* Rewrite the propositional logic statement of part a. into a logically equivalent statement using the logical connective 'or'.

c. *(0.5 mark)* Give the logical negation of the statement of part b. and distribute the negation using De Morgan Laws.

d. *(0.5 mark)* Translate the propositional logic statement of part c. into an English statement.

Question 6. *(2 marks)* Using a truth table: determine whether the following two statements are logically equivalent. Justify.

$$\neg A \to B$$

and

$$(\neg A \to \neg B) \iff B$$

Question 7. *(10 marks)* Using only the rules of inference and the rules of replacement show that the following argument is valid using Fitch style natural deduction:

$$P \to Q,\ \neg P \to R,\ (Q \vee R) \to S,\ \ \therefore S$$

Rules of Inference

$\to$**-elimination (or** $\to$**E)** *(or Modus ponens)*	$\Phi \to \Psi, \Phi \therefore \Psi$
Modus tollens (or MT)	$\Phi \to \Psi, \neg\Psi \therefore \neg\Phi$
$\leftrightarrow$**-introduction (or** $\leftrightarrow$**I)** *(or Biconditional introduction)*	$\Phi \to \Psi, \Psi \to \Phi \therefore \Phi \leftrightarrow \Psi$
$\leftrightarrow$**-elimination (or** $\leftrightarrow$**E)** *(or Biconditional elimination)*	$\Phi \leftrightarrow \Psi \therefore \Phi \to \Psi$ and $\Phi \leftrightarrow \Psi \therefore \Psi \to \Phi$
$\wedge$**-introduction (or** $\wedge$**I)** *(or Conjunction introduction)*	$\Phi, \Psi \therefore \Phi \wedge \Psi$
$\wedge$**-elimination (or** $\wedge$**E)** *(or Simplification)*	$\Phi \wedge \Psi \therefore \Phi$ and $\Phi \wedge \Psi \therefore \Psi$
$\vee$**-introduction (or** $\vee$**I)** *(or Disjunction introduction, Addition)*	$\Phi \therefore \Phi \vee \Psi$
Disjunction elimination (or DE)	$\Phi \to \Psi, \Theta \to \Psi, \Phi \vee \Theta \therefore \Psi$
$\vee$**-elimination (or** $\vee$**E)** *(or Disjunctive syllogism)*	$\Phi \vee \Psi, \neg\Phi \therefore \Psi$
Hypothetical syllogism (or HS)	$\Phi \to \Psi, \Psi \to \Theta \therefore \Phi \to \Theta$
Constructive dilemma (or CD)	$\Phi \to \Psi, \Theta \to \Pi, \Phi \vee \Theta \therefore \Psi \vee \Pi$
Destructive dilemma (or DD)	$\Phi \to \Psi, \Theta \to \Pi, \neg\Psi \vee \neg\Pi \therefore \neg\Phi \vee \neg\Theta$
Absorption (or ABS)	$\Phi \to \Psi \therefore \Phi \to \Phi \wedge \Psi$

Rules of Replacement

Associativity (or Asso.)	$\Phi\Box(\Psi\Box\Theta) \equiv (\Phi\Box\Psi)\Box\Theta$ where $\Box \in \{\wedge, \vee, \leftrightarrow\}$
Commutativity (or Comm.)	$\Phi\Box\Psi \equiv \Psi\Box\Phi$ where $\Box \in \{\wedge, \vee, \leftrightarrow\}$
Distributivity (or Dist.)	$\Phi \wedge (\Psi \vee \Theta) \equiv (\Phi \wedge \Psi) \vee (\Phi \wedge \Theta)$ and $\Phi \vee (\Psi \wedge \Theta) \equiv (\Phi \vee \Psi) \wedge (\Phi \vee \Theta)$
Double negation (or DN)	$\neg\neg\Phi \equiv \Phi$
De Morgan's laws (or DM)	$\neg(\Phi \vee \Psi) \equiv \neg\Phi \wedge \neg\Psi$ and $\neg(\Phi \wedge \Psi) \equiv \neg\Phi \vee \neg\Psi$
Transposition (or Trans.)	$\Phi \to \Psi \equiv \neg\Psi \to \neg\Phi$
Material implication (or MI)	$\Phi \to \Psi \equiv \neg\Phi \vee \Psi$
Biconditional implication (or BI)	$\Phi \leftrightarrow \Psi \equiv (\Phi \to \Psi) \wedge (\Psi \to \Phi)$
Exportation (or Expo.)	$(\Phi \wedge \Psi) \to \Theta \equiv \Phi \to (\Psi \to \Theta)$
Tautology (or Taut.)	$\Phi\Box\Phi \equiv \Phi$ where $\Box \in \{\wedge, \vee\}$