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Quiz 2

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §1.2 #4c *(3 marks)* Suppose that the augmented matrix for a system of linear equations has been reduced by row operations to the given row echelon form. Solve the system.

$$\begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 & \\ \end{matrix}\begin{bmatrix} 1 & -6 & 0 & 0 & 3 & -2 \\ 0 & 0 & 1 & 0 & 4 & 7 \\ 0 & 0 & 0 & 1 & 5 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Let $x_2 = s$, $x_5 = t$ $\quad s,t \in \mathbb{R}$

sub into

$x_1 - 6x_2 + 3x_5 = -2$

$x_3 + 4x_5 = 7$

$x_4 + 5x_5 = 8$

and solve for non free variables.

$\therefore$ $x_1 = -2 + 6s - 3t$

$x_2 = s$

$x_3 = 7 - 4t$ $\quad s,t \in \mathbb{R}$.

$x_4 = 8 - 5t$

$x_5 = t$

Question 2. §1.2 #11a *(3 marks)* Solve the following systems, where a, b are constants.

$$\begin{aligned} 2x + y &= a \\ 3x + 6y &= b \end{aligned}$$

$$\begin{bmatrix} 2 & 1 & a \\ 3 & 6 & b \end{bmatrix}$$

$\sim$ $2R_2 \to R_2$ $\begin{bmatrix} 2 & 1 & a \\ 6 & 12 & 2b \end{bmatrix}$

$\sim$ $-3R_1 + R_2 \to R_2$ $\begin{bmatrix} 2 & 1 & a \\ 0 & 9 & 2b - 3a \end{bmatrix}$

$\to$ $9R_1 \to R_1$ $\begin{bmatrix} 18 & 9 & 9a \\ 0 & 9 & 2b - 3a \end{bmatrix}$

$\sim$ $-R_2 + R_1 \to R_1$ $\begin{bmatrix} 18 & 0 & 12a - 2b \\ 0 & 9 & 2b - 3a \end{bmatrix}$

$\sim$ $\frac{1}{18}R_1 \to R_1$, $\frac{1}{9}R_2 \to R_2$ $\begin{bmatrix} 1 & 0 & (6a - b)/9 \\ 0 & 1 & (2b - 3a)/9 \end{bmatrix}$

$\therefore$ $(x, y) = ((6a - b)/9, (2b - 3a)/9)$

Question 3. §1.2 #32 *(4 marks)* Reduce

$$\begin{bmatrix} 2 & 1 & 3 \\ 0 & -2 & -29 \\ 3 & 4 & 5 \end{bmatrix}$$

to reduced row echelon form without introducing fractions at any intermediate stage.

$\sim$ $2R_3 \to R_3$ $\begin{bmatrix} 2 & 1 & 3 \\ 0 & -2 & -29 \\ 6 & 8 & 10 \end{bmatrix}$

$\sim$ $-3R_1 + R_3 \to R_1$ $\begin{bmatrix} 2 & 1 & 3 \\ 0 & -2 & -29 \\ 0 & 5 & 1 \end{bmatrix}$

$\sim$ $2R_3 \to R_3$ $\begin{bmatrix} 2 & 1 & 3 \\ 0 & -2 & -29 \\ 0 & 10 & 2 \end{bmatrix}$

$\sim$ $5R_2 + R_3 \to R_3$ $\begin{bmatrix} 2 & 1 & 3 \\ 0 & -2 & -29 \\ 0 & 0 & -143 \end{bmatrix}$

$\sim$ $\frac{-1}{143}R_3 \to R_3$ $\begin{bmatrix} 2 & 1 & 3 \\ 0 & -2 & -29 \\ 0 & 0 & 1 \end{bmatrix}$

$\sim$ $-3R_3 + R_1 \to R_1$, $29R_3 + R_2 \to R_2$ $\begin{bmatrix} 2 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

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$$\sim \xrightarrow{-\frac{1}{2}R_2 \to R_2} \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\sim \xrightarrow{-R_2 + R_1 \to R_1} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\sim \xrightarrow{\frac{1}{2}R_1 \to R_1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$